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THE THEORY OF THE OCULAR SPECTROSCOPE.

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THE forms of spectroscope in common use in astronomical spectroscopy may be classified under three general heads: (1) the compound slit spectroscope, (2) the objective spectroscope, and (3) the ocular (*sic*) spectroscope. Each class of instruments has its own peculiar merits and defects which determine the class of work to which it can be applied with best advantage. Thus, the compound slit spectroscope is best suited to determinations of absolute or relative wave-length, but is wasteful of light. The objective spectroscope is very efficient in this latter respect,[1] but in the usual form in which it is used, that of an objective prism, it is unsuited to accurate determinations of wave-length,[2] and is, moreover, very expensive.[3] The ocular

[1] "Theory of the Objective Spectroscope," ASTROPHYSICAL JOURNAL, 4, 56, June 1896.

[2] In some of the forms described by Professor Hale and the writer this objection may not apply. See paper on "The Objective Spectroscope," ASTROPHYSICAL JOURNAL, 4, 64–78, June 1896.

[3] The cost of the objective prism can be considerably decreased without loss of resolving power or efficiency by use of the sectional prism or the fluid prism with free surfaces described by the writer. See ASTROPHYSICAL JOURNAL, 4, 245, March 1895, and 4, 274, November 1896.

spectroscope is the least expensive of any and retains many of the advantages of the objective prism as respects light efficiency, but as it has been heretofore made and used it is inferior in definition and resolving power to either of the forms first mentioned. The object of the present paper is to investigate the cause of this inferiority, and to indicate the manner in which this type of instrument may be improved in this respect.

In the case of the ocular spectroscope the dispersing train is placed directly in the path of the cone of rays from the objective either just inside or just outside the focus. In the latter case the train may be placed either in front of or behind an eyepiece or an equivalent eye lens. It is therefore first of all necessary to investigate the effect of aberration due to the introduction of a dispersing system into the cone of rays from the objective. The amount of aberration thus produced will set a limit to the possible resolving power of the system. We will consider first the simplest case, in which the dispersing train consists of a single prism placed at minimum deviation.

The aberration produced in such a case has been already investigated by Lord Rayleigh, who finds[1] for the longitudinal aberration in the primary plane

$$\delta v = \text{longitudinal aberration} = \frac{3y \sin \phi (\mu^2 - 1)}{\mu^2 \cos^2 \phi'} , \qquad (1)$$

where y is the semi-linear aperture (measured along the face) of the prism, μ its index of refraction for the ray passing at minimum deviation, ϕ and ϕ' the angles of incidence of the axial pencil on the first and second prism faces respectively. The angular width of the resulting image of an infinitely thin slit under the best conditions of focus is therefore

$$\omega = \delta v \frac{\theta}{u} , \qquad (2)$$

where θ is the angular semi-aperture of the cone of rays and u is the distance of the apex of the cone from the prism face. Hence, since

$$\theta = \frac{y \cos \phi}{u} ,$$

[1] "Investigations in Optics with Special Reference to the Spectroscope," *Phil. Mag.*, 3d ser., 9, 40–49.

we have
$$\omega = \frac{3\theta^2(\mu^2 - 1)\sin\phi}{\mu^2 \cos\phi \cos^2\phi'} , \qquad (3)$$

The effect of this in a continuous spectrum would be to practically obliterate the images of absorption lines whose angular width was less than ω. In the case of a 60° prism of light flint glass, $\mu = 1.6$, $\phi' = 30°$ and $\phi = \sin^{-1} 0.8 = 53° 8'$.

Hence
$$\omega = 3.25\theta^2 . \qquad (4)$$

In the case of the objective spectroscope unaffected by aberration, the purity of the spectrum is equal to the theoretical resolving power R of the instrument.[1] This corresponds to an angular separation of the images of the two resolved lines equal to[2]
$$\Omega = \frac{4}{7} D\Delta\lambda + \frac{a^2}{D\Delta\lambda + a} , \qquad (5)$$
where D is the dispersion of the spectroscope train, $\Delta\lambda$ the "width" of the lines, and a the resolving power of the spectroscope aperture. For a single prism at minimum deviation[3]
$$D = \frac{2\sin\phi'}{\sqrt{1 - \mu^2\sin^2\phi'}} \cdot \frac{dn}{d\lambda} . \qquad (6)$$

For the case just considered $\phi = 30°$, $\mu = 1.6$,
$$\therefore D = \frac{5}{3} \cdot \frac{dn}{d\lambda} .$$

For light flint $\frac{dn}{d\lambda} \cong 1 \times 10^{-5}$ (where λ is expressed in tenth meters) and $D = 0.000017$. For solar lines the average "width" of the line is probably less the 0.1 tenth-meters, while for the broad hydrogen lines in star spectra the "width" is perhaps 1.0 tenth-meter or more.[4] Assuming further that the prisms

[1] "The Objective Spectroscope," ASTROPHYSICAL JOURNAL, **4**, June 1896. See p. 60.

[2] "On the Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL, **3**, 335, May 1896.

[3] "General Conditions Respecting the Design of Astronomical Spectroscopes," ASTROPHYSICAL JOURNAL, **1**, 55, January 1895.

[4] "On the Resolving Power of Telescopes and Spectroscopes for Lines of Finite Width," *Phil. Mag.*, 5th ser., **43**, 317. Also *Mem. Spec. Ital.*, **26**, 15, and *Wied. Ann.* **61**, 622 *et seq.*

have a clear aperture $a = 2y \cos \phi = 25$ mm we have

$$a \cong 0.00002$$

and

$$\Omega \begin{cases} = 0.000019 & \text{for } \Delta\lambda = 0.1 \\ = 0.000021 & \text{for } \Delta\lambda = 1 \end{cases} . \tag{7}$$

In order that the effect of aberration may not seriously affect the resolving power of the instrument, ω must not exceed $\frac{1}{2}\ \Omega$.[1] Hence we must have from (4) and (7)

$$\theta^2 > 0.0000031 \quad \text{or} \quad \theta \gtrless 0.00176$$

i. e., the angular aperture of the cone of rays must not be larger than $\frac{1}{2\,8\,0}$, if the effect of aberration is not to be prejudicial to the resolving power of a single 60° prism of 1 inch clear aperture. The angular apertures of astronomical telescopes are very much larger than this, and the use of the prism in the direct cone of rays from the objective is therefore inadmissible, if apertures of any size are to be made use of.

Conversely, if the angular aperture of the telescope be given, we can easily determine the maximum permissible aperture and resolving power of a prism placed in the cone of rays. For astronomical telescopes θ varies from 0.035 to 0.025. Assume $\theta = 0.03$. From (4) and (5) we have at once

$$0.0029 = \frac{2}{7} D\Delta\lambda + \frac{a}{2\left(\frac{D\Delta\lambda}{a} + 1\right)} . \tag{8}$$

From what has preceded it will be at once evident that if θ is to be increased, we can only preserve definition by decreasing D or increasing a. In either case the quantity $\frac{D\Delta\lambda}{a}$ will become small, and may, in the denominator of the second term, be neglected. Hence, we have at once as a limiting relation between D and a

$$a_{\max.} \cong 0.006\,(1 - 100\, D\Delta\lambda) . \tag{9}$$

Hence, for a single 60° prism of light flint for which $D\Delta\lambda = 0.000017$ to 0.0000017, we must have

$$a \lessgtr 0.00599 \quad \text{or} \quad a \cong 20' .$$

[1] "Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL, 3, May 1896. See p. 336.

That is, the aperture of the prism must be considerably less than 0.1 mm.

Somewhat more favorable conditions may be realized by decreasing ϕ' or μ, or both. For with small values of ϕ', and therefore of ϕ, the aberration diminishes very nearly as $\sin \phi$, while, as seen from (6), the dispersion, and therefore the resolving power, diminishes somewhat less rapidly. We can find the maximum permissible value of r, the resolving power, by expressing directly the necessary ratio between ω and Ω, as defined by (3), (5), and (6). This gives

$$\frac{6\theta^2(\mu^2-1)\sin\phi}{\mu^2\cos\phi\cos^2\phi'} = \frac{4}{7}\frac{2\sin\phi'}{\sqrt{1-\mu^2\sin^2\phi'}}\cdot\frac{dn}{d\lambda}\cdot\Delta\lambda + \frac{a}{\frac{D\Delta\lambda}{a}+1}$$

or

$$3\theta^2\frac{\mu^2-1}{\mu}\cdot\frac{1}{\cos^2\phi'} = \frac{4}{7}\cdot\frac{dn}{d\lambda}\Delta\lambda + \frac{a}{D}\frac{dn}{d\lambda}\left(\frac{1}{\frac{D\Delta\lambda}{a}+1}\right). \tag{10}$$

From the well-known relation[1]

$$r = \frac{D\lambda}{a}$$

we get at once by substitution and transposition

$$\frac{1}{\frac{r\Delta\lambda}{\lambda}+1}\cdot\frac{\lambda}{r}\left(\frac{dn}{d\lambda}\right) = 3\theta^2\frac{\mu^2-1}{\mu}\cdot\frac{1}{\cos^2\phi'} - \frac{4}{7}\frac{dn}{d\lambda}\cdot\Delta\lambda\ , \tag{11}$$

or neglecting as before the quantity $\frac{r\Delta\lambda}{\lambda}$,

$$\frac{1}{r} \cong \frac{1}{\lambda}\left[3\theta^2(\mu-\mu^{-1})\frac{1}{\cos^2\phi'\frac{dn}{d\lambda}} - \frac{4}{7}\Delta\lambda\right], \tag{12}$$

which shows, as already stated, that as ϕ' decreases, r increases, although not very greatly.

Assuming the values of λ, $\frac{dn}{d\lambda}$, $\Delta\lambda$ (max. = 1 tenth-meter) already used, and taking θ as 0.03, we have for r

$$\frac{1}{r} \cong \frac{1}{5600}\left(\frac{270}{\cos^2\phi'} - \frac{4}{7}\right).$$

[1] *Loc. cit.*

For the maximum value of $\cos \phi' = 1$ (which is, of course, unattainable, we would therefore have for r

$$r \cong 20$$

or the closest lines that could be resolved would be separated by an interval of about 300 tenth-meters — over fifty times the distance between the two D lines. Such an arrangement is, therefore, inadmissible for work in the visible part of the spectrum.[1]

For particular portions of the spectrum the amount of aberration may be considerably reduced by the use of a direct vision prism instead of a simple prism. As in the case of the achromatic objective, we may, by properly proportioning the angles of the different component prism surfaces, completely correct for aberration for one wave-length λ_0 in the spectrum. The definition can, however, only be good over a very limited range of spectrum; all other parts will suffer from the effect of aberration to an extent depending, as in the case of the simple prism, on the dispersion and aperture, *i. e.*, on the resolving power, of the prism system. The exact effect of aberration on a cone of rays passing through the combination of prisms necessary to form the direct vision system can be determined in a manner similar to that employed in the case of a simple prism, but in the case of a combination of five prisms such as is commonly used it becomes very complicated. A simpler, and at at the same time fairly approximate, method of determining its effect is to consider such a system as optically equivalent to a simple prism of the same aperture, and of a refracting angle such as will produce a deviation of the rays under consideration equal to the difference in deviation between these rays and the rays of wave-length λ_0, for which the aberration is zero in the direct vision system. Thus, with a dispersion of 2° between C

[1] This is not the case in work in the infra-red with the spectrobolometer, where λ is very large and θ is small. In such a case, a collimator may be dispensed with, not only without detriment, but with advantage, because the loss of energy due to absorption in the collimator lens is thus avoided. See article on "Fixed Arm Spectroscopes," *Phil. Mag.*, **38**, October 1894; also *Report of the Smithsonian Astrophysical Observatory*, 1893.

and E, the aberrational effects on rays of wave-length λ_g (assumit to be zero for wave-length λ_c) will be the same, to the above degree of approximation, as is produced by a prism of an index μ and a refracting angle $2\phi'$ equal to

$$\frac{2}{\mu - 1},$$

since the equivalent prism is thin.

For $\mu = 1.6$ as assumed before

$$2\phi' = 3^\circ\ 20' \quad \text{and} \quad \phi = 2^\circ\ 40', \qquad (13)$$
$$\therefore \omega \cong 0.085\theta^2,$$

or only about one thirty-seventh as great as with the 60° prism already considered.

The dispersing power of such a system, $D = \frac{d\theta}{d\lambda}$, is about equal to two 60° prisms of the same index $\mu = 1.6$. The effect of aberration on a system of given resolving power, $Da = Const.$, is, therefore, very considerably less than when simple prisms are used, and we may, therefore, use apertures considerably larger than is possible in the latter case without injury to definition. Even the very considerable gain thus resulting is not, however, sufficient to render the use of any considerable resolving power possible, as for the value of θ assumed before we find at once from (13) and (5)

$$a_{max.} = 0.00015 - 0.58\, D\Delta\lambda \cong 0.00013, \qquad (14)$$
$$\cong 26'',$$

or b, the limiting linear aperture of the prism train, is about 4 mm, corresponding to a resolving power in the prism train of 1360 units, a power barely sufficient to resolve, but not distinctly separate, the D lines under the best conditions.

As in the preceding case, something more may be gained by diminishing the dispersive power of the combination and increasing the aperture, but the gain will not be nearly so great, because the possible range in the value of ϕ and ϕ' is much

smaller. The maximum value of r will, as before, be found by equating (3) and (5). This gives as before

$$\frac{6\theta^2(\mu^2-1)\sin\phi'}{\mu\cos\phi\cos^2\phi'} = \frac{4}{7}D\Delta\lambda + a\left(\frac{1}{\frac{D\Delta\lambda}{a}+1}\right).$$

But in this case, by our assumption of equivalent prisms,[1]

$$\phi' = (\mu-1)\frac{\delta}{2} = \frac{\mu-1}{2}\cdot D(\lambda_C-\lambda_E)\ , \qquad (15)$$

and since ϕ', as just shown, is small for the most favorable case, we have by substitution and reduction

$$3\theta^2\frac{(\mu^2-1)(\mu-1)}{\mu}(\lambda_C-\lambda_E) = \frac{4}{7}\Delta\lambda + \frac{\lambda}{r}\left(\frac{1}{\frac{r\Delta\lambda}{\lambda}+1}\right), \qquad (16)$$

or, assuming the quantity $\frac{r\Delta\lambda}{\lambda}$ to have a maximum value 0.5, we get

$$\frac{1}{r} \cong \frac{1}{\lambda}\left[4.5\theta^2(\mu-\mu^{-1})(\mu-1)(\lambda_C-\lambda_E) - \frac{6}{7}\Delta\lambda\right], \qquad (17)$$

and assuming the values of λ, θ, and $\Delta\lambda$ (max.= 1 tenth-meter) used before, we get

$$r_{max.} \cong \frac{5600}{3.13-\frac{6}{7}} \qquad (18)$$

$$r_{max.} \cong 2450\ ,$$

about 120 times that obtainable with a simple prism, and nearly equal to that obtainable with a single 60° prism of white flint of about 15mm aperture.

As the value of θ increases the value of $r_{max.}$ rapidly decreases. Thus for a reflecting telescope $\theta = 0.09$ we would have

$$r_{max.} = \frac{5600}{27.13} \cong 200\ . \qquad (19)$$

It is interesting to compare the effects of aberration due to the direct vision prism system just considered with that due to a simple lens. From (13) we have for the E line

$$\omega = 0.085\,\theta^2\ . \qquad (20)$$

[1] Assuming the dispersion uniform throughout the range λ_C to λ_E.

The longitudinal aberration in a simple lens having faces of the most favorable curvatures is $0.83u\theta$. The angular width ω' of the image of a line in the best focus is therefore

$$\omega' = 0.83\,\theta^3 \ . \qquad (21)$$

Hence for a value of $\theta = 0.03$ we have

$$\frac{\omega}{\omega'} = 3\frac{1}{3} \ ,$$

or the effect of the prism is over three times as great as the effect of a simple lens of glass. For the portion of the spectrum above E the effect of aberration becomes rapidly worse; at G, for example, the effect will be about six times as great in the case of the direct vision prism as in the case of the lens.

In view of these results it is evident that the general statement made by Scheiner[1] and others that when direct vision prism systems are placed directly in a cone of rays from the telescope objective, the loss in definition is "slight," and that "the errors resulting from imperfect parallelism of the rays are of about the same order of magnitude as those which result from disregarding the thickness in the treatment of simple lenses," are incorrect, or at least incomplete and therefore misleading. It is only true for very small resolving powers which would be practically useless in most lines of modern spectroscopic and spectrographic research. Secchi used this form of instrument, but only recommended it for small apertures.[2] He soon discarded it for the objective prism of 16 cm aperture which, he states, gave spectra which were both far brighter and far better defined than those obtained with the ocular spectroscopes he first used.[3]

The preceding considerations apply to the ocular spectroscope whether the prism system be placed inside or outside the focus. In the latter arrangement some advantage may be gained in the way of diminishing aberration by placing the prism system outside the eye lens of the telescope, as the angu-

[1] *Astronomical Spectroscopy* (Frost's translation), p. 30.

[2] *C. R.* **65**, 389 and 979. [3] *C. R.* **69**, 1054.

lar divergence of the cone of light, *i. e.*, the value of θ in (17) is here considerably less than that of the main cone of rays inside the focus. It is obvious, however, that this advantage is secured only at the sacrifice of resolving power (owing to the necessarily small dimensions which the prism system must then assume), and at the sacrifice of another important advantage, *i. e.*, the possibility of using the system either in micrometric or spectrographic work in the principal focal plane.

In the preceding investigation the effect of the thickness of the prism system has not been considered. The effect of this will be to increase the aberration to an extent depending on the distance of the system from the focal plane. When the prism is placed inside the focus this distance may be made large in comparison with the thickness and the effect of the latter element will be small. But when the prism system is placed outside the eye lens, and thus of necessity near the vertex of the cone of rays, the magnitude of the effect is increased. In this respect, therefore, the latter form of ocular spectroscope is inferior (in theory at least) to the one first considered.

To sum up briefly, it is evident from the preceding results: (1) That the use of simple prisms of any angle directly in the cone of rays from the objective is only possible when the aperture is so small as to be practically useless for the large majority of spectroscopic investigations. (2) That the effect of aberration due to the prism system may be reduced and the use of somewhat larger apertures rendered possible by the use of direct vision prism systems instead of simple prisms, but that this gain is not large and is confined to a comparatively narrow range of spectrum, and is secured moreover only at a great sacrifice of light efficiency, especially in the upper part of the spectrum, owing to the greatly increased absorption of the direct vision combination, the flint glass element of which must necessarily be quite dense or quite thick. (3) That aberration can also be reduced by placing the prism system outside the eyepiece (or what amounts to the same thing, a single eye lens), but that this improvement is secured by a still further sacrifice of resolving power, and

what is still worse, a sacrifice of the possibility of using the system in focal plane work.

The general conclusion to be drawn is, that in its present form the ocular spectroscope is unsuited to, and incapable of satisfying the demands of modern spectroscopic and spectrographic work. I shall show in a subsequent paper how this type of instrument may be so modified as to be greatly improved in optical performance and applied with advantage to the investigations of a number of interesting and important researches in astronomical spectroscopy and spectral photometry.

ALLEGHENY OBSERVATORY,
1899.

DESCRIPTION OF A NEW TYPE OF FOCAL PLANE SPECTROSCOPE AND ITS APPLICATION TO ASTRONOMICAL SPECTROSCOPY.

By F. L. O. WADSWORTH.

IN the preceding paper I have investigated the effect of aberration in limiting the resolving power of the ocular spectroscope as usually constructed, and have shown that this limitation is so serious as to prohibit the use of this form of instrument in the great majority of cases. The advantages of this type of spectroscope over the compound slit spectroscope, on the score of light efficiency, and over both the compound spectroscope and the objective spectroscope, on the score of greatly diminished first cost, are, however, so great that I was induced to devote my attention to the question of overcoming or avoiding the difficulties due to aberration without sacrificing resolving power. In the course of the preceding investigation a method of doing this suggested itself which is extremely simple and effective, but which does not seem to have been previously used. Referring to equation (3) of the preceding paper, it is obvious that the only quantities in that equation which can be varied to any degree are ϕ, ϕ', and θ. As was there pointed out, a most decided improvement can be effected by decreasing the value of θ; but we are limited in this direction by mechanical considerations, as it is impossible to indefinitely, or even very greatly, increase the focal length of our present equatorial telescopes. But there is another very simple way of accomplishing the same result, and this is to insert in front of the prism system a negative collimator lens. By making the latter of suitable focal length, the angular divergence of the cone of rays passing through the prism system may be made as small as we please, and, over a considerable range of wave-length, reduced to zero. It is then only necessary to place behind the system a second positive lens of whatever focal length desired to form the spectral image. The form and con-

struction of these lenses will be determined by the angular aperture of the objective and the nature of the color curve. Simple lenses are, of course most desirable if they can be employed without themselves introducing an undue amount of either spherical or chromatic aberration. We will first examine the conditions which determine the former requirement.

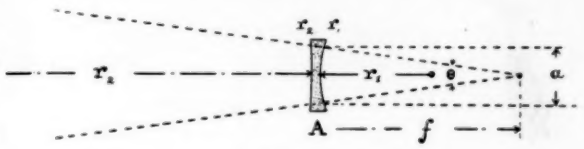


FIG. 1.

The form of simple lenses which introduces the minimum amount of spherical aberration is defined by the relations

$$\frac{1}{r_1} = \frac{1}{2(\mu-1)(\mu+2)}\left[(2\mu^2+\mu)\frac{1}{f''} + (2\mu^2-\mu-4)\frac{1}{f'}\right], \qquad (1)$$

$$\frac{1}{r_2} = \frac{1}{2(\mu-1)(\mu+2)}\left[(2\mu^2+\mu)\frac{1}{f'} + (2\mu^2-\mu-4)\frac{1}{f''}\right], \qquad (2)$$

where r_1 and r_2 are the radii of curvature of the first and second surfaces, and f', f'', the conjugate focal distances of the pencil of rays traversing the lens. If we make the focal length of the lens such that the pencil of light under consideration is rendered parallel, or, what amounts to the same thing, if we place the lens at its own focal distance from the focal plane of the objective, we have at once

$$\frac{1}{f'} = 0 \quad \text{and} \quad \frac{1}{f''} = \frac{1}{f}.$$

Then

$$\frac{r_2}{r_1} = \frac{2\mu^2+\mu}{2\mu^2-\mu-4}, \qquad (3)$$

the well-known relation defining the ratio of the radii of curvatures of the so-called "crossed lens," the face of maximum radius of curvature, r_2, being presented to the incident cone of light from the large objective, as in Fig. 1.

Under the above conditions the expression for the longitudinal aberration becomes[1]

[1] LORD RAYLEIGH, "On the Minimum Aberration of a Single Lens for Parallel Rays," *Proc. Camb. Phil. Soc.*, **8**, 373, April 1880.

$$\delta\nu = -\frac{a^2}{32f}\,\mu\left[\frac{4\mu - 1}{(\mu - 1)^2(\mu + 2)}\right], \tag{4}$$

where a is the aperture of the objective. The value of $\delta\nu$ decreases as μ increases. It is, however, unadvisable to employ very dense glass, on account of the relatively greater absorption and loss by reflection at the refracting surfaces. If we assume $\mu = 1.6$, corresponding to the ordinary light flint, we have

$$\delta\nu = 0.21\,\frac{a^2}{f}\,. \tag{5}$$

If we assume, as before, that in order to avoid injuring the definition the effect of the longitudinal aberration must not exceed one-half the resolving power of the aperture, then

$$\frac{\delta\nu\theta}{f} \leqq \frac{\lambda}{b} \quad \text{or} \quad \delta\nu \leqq \frac{\lambda}{\theta^2} = \frac{4\lambda}{a^2}f^2\,. \tag{6}$$

From (5) and (6) we have at once, for the limiting relation between θ and f,

$$\theta \leqq \sqrt[4]{\frac{\lambda}{0.83f}}\,. \tag{7}$$

If, as before, we assume $\theta = 0.03$, which corresponds to the usual construction of refracting object-glasses, we have for f

$$\begin{aligned} f_{\max.} &= \frac{\lambda}{0.67} \times 10^8\,, \\ &= 83.6\,\text{cm} \quad \text{for} \quad \lambda_\nu = 5600 \text{ tenth-meters}\,, \\ &= 64.2\,\text{cm} \quad \text{for} \quad \lambda_p = 4300 \text{ tenth-meters}\,. \end{aligned} \tag{8}$$

That is, for an ordinary telescope the focal length of the negative collimator must not exceed 84 cm for the visual rays, or 64 cm for the photographic rays. This condition at once determines the possible linear aperture a, and hence the resolving power of a given prism train. For, under the conditions above imposed,

$$\begin{aligned} a &= 2f\theta\,, \\ &= 5.02\,\text{cm} \quad \text{for visual rays}\,, \\ &= 3.85\,\text{cm} \quad \text{for photographic rays}\,. \end{aligned} \tag{9}$$

The possible maximum values of f and a decrease very rapidly

as the value of θ increases. For photographic telescopes and for reflecting telescopes the value of θ is frequently as large as 0.10. For this value the limiting values of f and a for a single lens become

$$\begin{aligned} f_{\max.} &\cong 0.8\,\text{cm} \;;\quad a_{\max.} \cong 1.6\,\text{mm} \quad \text{for } \lambda = 5600\,, \\ f_{\max.} &\cong 0.6\,\text{cm} \;;\quad a_{\max.} \cong 1.2\,\text{mm} \quad \text{for } \lambda = 4300\,. \end{aligned} \tag{10}$$

For such cases, therefore, the possible dimensions of a single lens used as a negative collimator alone become very small.

If, however, we impose certain conditions with reference to the second positive lens used behind the prism train to form the spectral image, we may considerably reduce the effects of aberration due to the first lens alone, and thus considerably increase the possible aperture of the simple lens system. Thus, if we agree to make this second lens of the same material and focal length as the first (as may often be done) the aberration in the two lenses will be equal but of opposite sign, and, if the two were in contact, the resultant effect would be zero. The compensation will not be quite exact, on account of the separation of the two lenses and the intervention of the intermediate prism train, but the residual effect due to the latter will be small, and may, up to apertures equal to those already considered, be safely neglected. With such a construction as indicated above we may, therefore, safely employ single lenses up to 5 cm or more in diameter, even when the angular aperture of the objective is as large as 1 to 5, provided it has no large amount of chromatic aberration.

When the large objective itself has a considerable amount of chromatic aberration the use of a single lens is not, however, so satisfactory, as it is only possible then to collimate the rays over a limited range of wave-lengths. In order to increase the range over which the spectrum may be simultaneously brought to a sharp focus by the second positive lens we must construct the collimator so as to correct, as far as possible, for the chromatic aberration of the large objective. The degree to which this result may be secured will be next investigated.

In Fig. 2 let $AF = d =$ distance of the negative collimator from focal plane of the visual rays, $\lambda \cong 5600$; let $AF' = d' =$ distance of the negative collimator from focal plane for photographic rays, $\lambda \cong 4300$.

For the focal length of the negative lens we have the usual formulae

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \left(\mu_c - 1\right)\left[\frac{1}{r_1} + \frac{1}{r_2}\right] + (\mu_f - 1)\left[\frac{1}{r_3} + \frac{1}{r_4}\right] , \quad (11)$$

f_1, f_2, being the individual focal lengths of the component flint and crown elements and r_1, r_2, r_3, r_4, the radii of curvatures of the successive surfaces measured from the right toward the left.

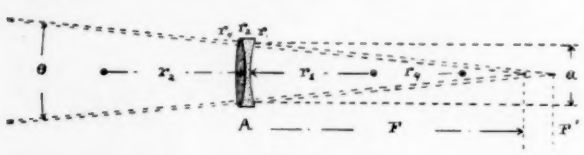


FIG. 2.

In order to render both the visual and photographic rays, λ_v and λ_p, simultaneously parallel, we must have

$$\frac{dF}{d\lambda}(\lambda_v - \lambda_p = \Delta\lambda) = d' - d = \Delta F ,$$

which gives at once

$$d' - d = \left\{ \frac{d\mu_c}{d\lambda}\left[\frac{1}{r_1} + \frac{1}{r_2}\right] + \frac{d\mu_f}{d\lambda}\left[\frac{1}{r_3} + \frac{1}{r_4}\right] \right\} F^2 (\lambda_v - \lambda_p) ,$$

or

$$\frac{\Delta F}{F^2} = \frac{d\mu_c}{d\lambda}(\Delta\lambda)\frac{1}{(\mu_c - 1) f_1} + \frac{d\mu_f}{d\lambda}(\Delta\lambda)\frac{1}{(\mu_f - 1) f} , \quad (12)$$

For the usual soft crown and dense flint glasses we have

$$\left.\begin{aligned} \mu_c &= 1.5163 \quad \text{for} \quad \lambda_v \\ \mu_f &= 1.6256 \quad \text{for} \quad \lambda_v \end{aligned}\right\} (\lambda = 5600) .$$

Also

$$\frac{d\mu_c}{d\lambda} \cdot \frac{\Delta\lambda}{(\mu_c - 1)} = \frac{\delta\mu_c}{\mu_c - 1} \cong 0.01995$$

$$\frac{d\mu_f}{d\lambda} \cdot \frac{\Delta\lambda}{(\mu_f - 1)} = \frac{\delta\mu_f}{\mu_f - 1} \cong 0.03133 .$$

Introducing these values and solving for f_1 and f_2 we get

$$\frac{1}{f_1} = \frac{1}{F}\left(87.87\frac{\Delta F}{F} + 2.753 + \right)$$
$$-\frac{1}{f_2} = \frac{1}{F}\left(87.87\frac{\Delta F}{F} + 1.753 + \right) . \qquad (13)$$

When ΔF is zero these equations become of course the usual formulae for a compound lens achromatic for the two rays λ_v and λ_β.

As a practical example let us consider the construction of a negative collimator of 3 cm aperture for the Yerkes 40-inch telescope. In this case

$$\theta \cong \tfrac{1}{2}\left(\frac{1}{19}\right),$$
$$\therefore F = 3 \times 19 = -57\,\text{cm} ,$$
$$\Delta F = -6\,\text{cm}\ ^1 .$$

With these values we obtain at once

$$f_1 = -\frac{F}{12} = -4.75\,\text{cm}$$
$$f_2 = +\frac{F}{11} = +5.18\,\text{cm} , \qquad (14)$$

i. e., the desired chromatic collimation will be secured by using a negative lens of crown glass of 4.75 cm (virtual) focal length in combination with a positive lens of flint glass of a focal length of 5.18 cm.

In order to determine the best radii of curvature of the successive surfaces we have, as the condition of aplanatism for parallel rays,

$$\frac{m_1}{f_1}\left(\epsilon^2 + \frac{n_1}{f_1^2}\right) + \frac{m_2}{f_2}\left[\epsilon'^2 - \frac{1}{f_1}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \frac{n_2}{f_2^2}\right] = 0 , \qquad (15)$$

where m_1, n_1, m_2, n_2, are constants defined by the relations

$$m_1 = \frac{\mu_c}{\mu_c + 2} \qquad m_2 = \frac{\mu_f}{\mu_f + 2}$$
$$n_1 = \frac{4\mu_c - 1}{4(\mu_c - 1)^2} \qquad n_2 = \frac{4\mu_f - 1}{4(\mu_f - 1)^2} . \qquad (16)$$

[1] From an unpublished determination of the color curve of the 40-inch objective made by the writer in 1897.

If we assume, as a second condition, that the inner faces of the two lenses shall be cemented (to avoid loss of light), then

$$r_2 = r_3 ,$$

and we have, as a second equation,

$$\frac{q_1}{f_1} + m_1\epsilon = \frac{q_2}{f_1} + p_2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + m_2\epsilon' , \tag{17}$$

where

$$\begin{aligned} q_1 &= \frac{2\mu_c^2 - \mu_c - 4}{2(\mu_c - 1)(\mu_c - 2)} \\ p_1 &= \frac{2\mu_c^2 + \mu_c}{2(\mu_c - 1)(\mu_c + 2)} , \end{aligned} \tag{18}$$

and q_2 and p_2 have similar values in terms of μ_f.

If we assume as before the usual soft crown and dense flint construction, we have for μ_c and μ_f

$$\left.\begin{aligned} \mu_c &= 1.5163 \\ \mu_f &= 1.6256 \end{aligned}\right\} \text{ for } \lambda_D = 5600 .$$

Substituting these values in (16) and (18), and solving (15) and (17) for ϵ and ϵ' we find:

$$\begin{aligned} p_1 &= \ \ 1.6840 & p_2 &= \ \ 1.5324 \\ q_1 &= -0.2528 & q_2 &= -0.0750 \\ m_1 &= \ \ 0.4312 & m_2 &= \ \ 0.4484 \\ m_1 &= \ \ 4.7498 & m_2 &= \ \ 3.5146 , \end{aligned}$$

and for ϵ and ϵ'

$$\epsilon = +0.3148 \qquad \epsilon' = +0.4444 .$$

For the radii of curvature of the four surfaces we have

$$\begin{aligned} \frac{1}{r_1} &= \frac{p_1}{f_1} + m_1\epsilon & \text{or} \quad r_1 &= -4.56 \\ \frac{1}{r_2} &= \frac{1}{r_3} = \frac{q_1}{f_1} + m_1\epsilon & r_2 &= r_3 = +5.305 \\ \frac{1}{r_4} &= \frac{p_2}{f_1} + q_2\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + m_2\epsilon' & r_4 &= -8.326 . \end{aligned}$$

The crown lens is therefore a double concave and the flint lens a double convex form, and the crown lens is turned toward the prism train as in Fig. 2.

When a compound lens of the form above described is used

as a collimator we may without restriction use a lens of any material and focal length desired for the image-forming lens beyond the prism train. If simple lenses are used for this purpose the limiting apertures and focal lengths which it is possible to employ are determined by equations (7) and (9) as before. In most cases simple lenses answer very satisfactorily in this place for the reason that the angular aperture θ_1 of this lens may be made small, thus reducing spherical aberration, and chromatic aberration need not be considered in the formation of a spectral image.

The first instrument of this kind which I constructed and tested was a small direct prism form, designed at the request of Professor Stone for the 26-inch Leander McCormick telescope of the University of Virginia.[1] The mechanical and optical construction is shown in detail in the cross-sectional view, Figs. 1 and 2, Plate I. *A* is the negative collimating lens, which in this case is of 16 mm clear aperture and 76 mm focal length. *B* is the direct-vision prism system, consisting of 5 prisms as shown, with a clear aperture of 13 mm and a total dispersion, C to H, of about 6°. *C* is the positive image-forming lens, which in this case is of the same focal length, 76 mm, as the negative collimator. *D* is a scale in the focal plane, and *E* and *F* the plane parallel glass reflector and lamp for illuminating the latter. *G* is an ordinary positive eyepiece of about 45 mm equivalent focus.

The negative collimator *A* and prism system *B* are mounted together in a short tube, *b*, which slips tightly into a second tube, *c*, carrying the positive lens, *C*. The tube *c* slides rather tightly in the third tube *d* which fits into the draw tube of the great telescope. The tube *d* carries at one end a bracket which supports the lamp box, *f*, and is threaded at the outer end to receive the fourth eyepiece tube, *g*. A fifth tube, *e*, fitting tightly into *g*, carries the reflector and scale. The tubes *g* and *e* are slotted at the point *i* to receive a metal diaphragm or slide, *k*.

The instrument is adjusted before being attached to the large

[1] See my *Report of the Allegheny Observatory*, 1898–9, p. 9.

telescope, as follows: The tube *b*, containing the negative collimator and prism system, is first removed and the eyepiece is focused on the scale *D*. The positive lens *C* is then focused for parallel rays by means of the tube *c*, either by looking at a distant object, or better, by placing the instrument in front of the object glass of a transit and focusing on the cross-wires of the same. The tube *b* is then replaced, taking care not to disturb the relative positions of *c* and *d*. The whole instrument is then placed in the draw tube of the large telescope and focused on any particular line in the spectrum by moving the latter in or out as with an ordinary eyepiece.

It is evident that with this form and construction of instrument, micrometric and wave-length observations and comparisons may be made directly at the focal plane by the aid of the scale *D* in the same manner as with the objective spectroscope. The instrument is also adapted to photographic work. To carry this out the eyepiece is removed and a small photographic plate substituted, the sensitive surface of the latter being held against the scale *D* by means of the wooden plug and spring *L*. The diaphragm *k* then serves as a slide, and the tube *g* with diaphragm *k* and plug *L* as a plate-holder, which is readily detached from the instrument by unscrewing it from *d*. The spectrum of the object under examination and the reference scale *D* can thus be photographed simultaneously on the sensitive plate.

The optical resolving power of the instrument above described is small, being about equivalent to one 60° white flint prism of 1-inch clear aperture, a power about three times that required to separate the D lines.

In order to subject it to a more critical and severe test than that involved in the examination of stellar spectra, the arrangement shown in Fig. 3 of the same plate was used. In this figure *M* is a large collimator (7 cm aperture) with slit at *s* accurately adjusted for parallel rays by the usual method. *N* is an ordinary observing telescope with the above described instrument inserted at *P* in the place of the usual eyepiece. When the slit *s* is illuminated by sunlight an image of the solar spec-

PLATE I.

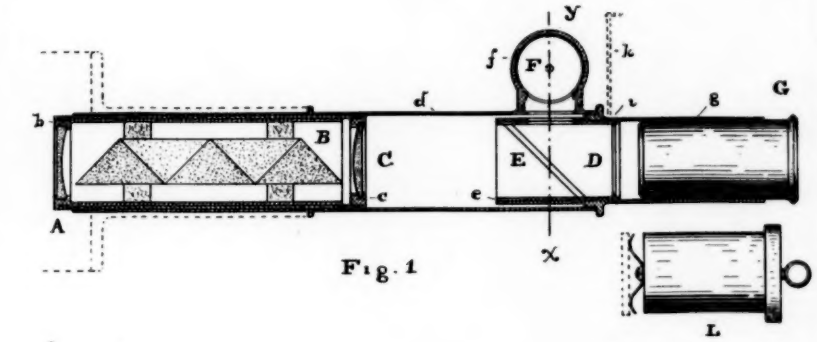


Fig. 1

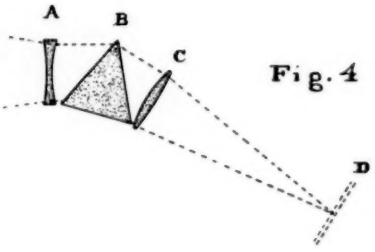


Fig. 4

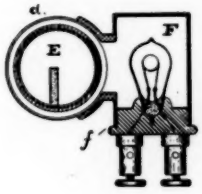


Fig. 2

Section on X. Y.

Fig. 3

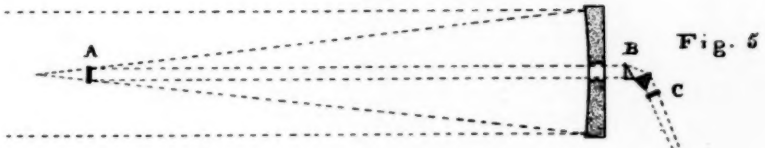
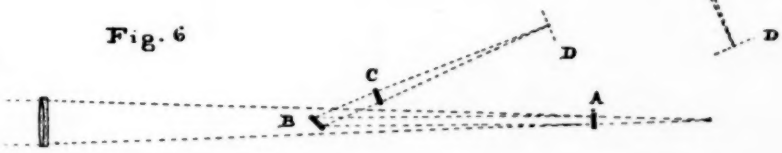


Fig. 5

Fig. 6



trum is, of course, formed at the focal plane of the instrument.

The definition of the prism train alone can be tested by removing it from the instrument and placing it at R, between the two telescope objectives, as in the usual spectroscope. Tested carefully in this way the definition and resolving power was found to be equally good with both arrangements. The nickel line between the D's was sharp and well resolved. When, however, the negative collimator lens and positive lens, C, was removed and the same direct-vision prism alone used at P, as in the usual form of ocular spectroscope, all the lines became very hazy and ill defined, and it was found impossible to separate even the two D lines.

The second instrument of this new form which has been constructed was one for the Crossley 36-inch reflector of the Lick Observatory. Professor Keeler, just before his death, had planned to use an ocular spectroscope consisting of a single 50° prism of quartz placed just inside the focus on the Crossley reflector, for the purpose of photographing the spectra of faint stars. As was shown in my previous paper, such an arrangement would be very bad optically, and this was found by experiment to be the case. The idea of using a negative collimator to improve the definition then independently suggested itself to Professor Campbell, who entered into correspondence with Mr. Brashear in regard to the matter in September 1900.

In the case of the Crossley reflector the ratio of aperture to focal length is 3 : 17.5 or 1 : 5.84. Hence $\theta = 0.086$.

From (7) we have, therefore, in this case,

$$f_{\max.} = \frac{\lambda \times 10^6}{45},$$

$= 1.25\,\text{cm}$ for the visual rays, $\lambda = 5600$,

$= 0.95\,\text{cm}$ for the photographic rays, $\lambda = 4300$.

The permissible aperture of the negative collimator therefore is $a = \frac{f}{5.83} \cong 2\,\text{mm}$ for the photographic region of the spectrum.

It was desired to use an aperture about twelve times as large as this, the aperture of the quartz prism being about 1 inch.

In order to make this possible without introducing compound lenses, the plan described in an earlier paragraph of making the second positive lens of the same material and focal length as the negative collimator was adopted. As the large objective was itself perfectly achromatic (being a reflector), chromatic aberration did not have to be considered. In order to avoid absorption the lenses were made of quartz (like the prism), the plane of the lens being cut perpendicular to the axis of the crystal (to avoid double refraction).

The dimensions and computed radii of the lenses were as follows:

$$\left.\begin{array}{ll} \text{negative collimator} & f = -152.4\,\text{mm (6 in.)} \\ \text{positive lens} & f = +152.4\,\text{mm (6 in.)} \end{array}\right\} \text{for G .}$$

For G ($\lambda = 4307$) μ_o, the ordinary index of quartz, is 1.55425. Hence for a "crossed" lens we have from (3)

$$r_1 \cong 9r_2 ,$$

and with this relation we find at once

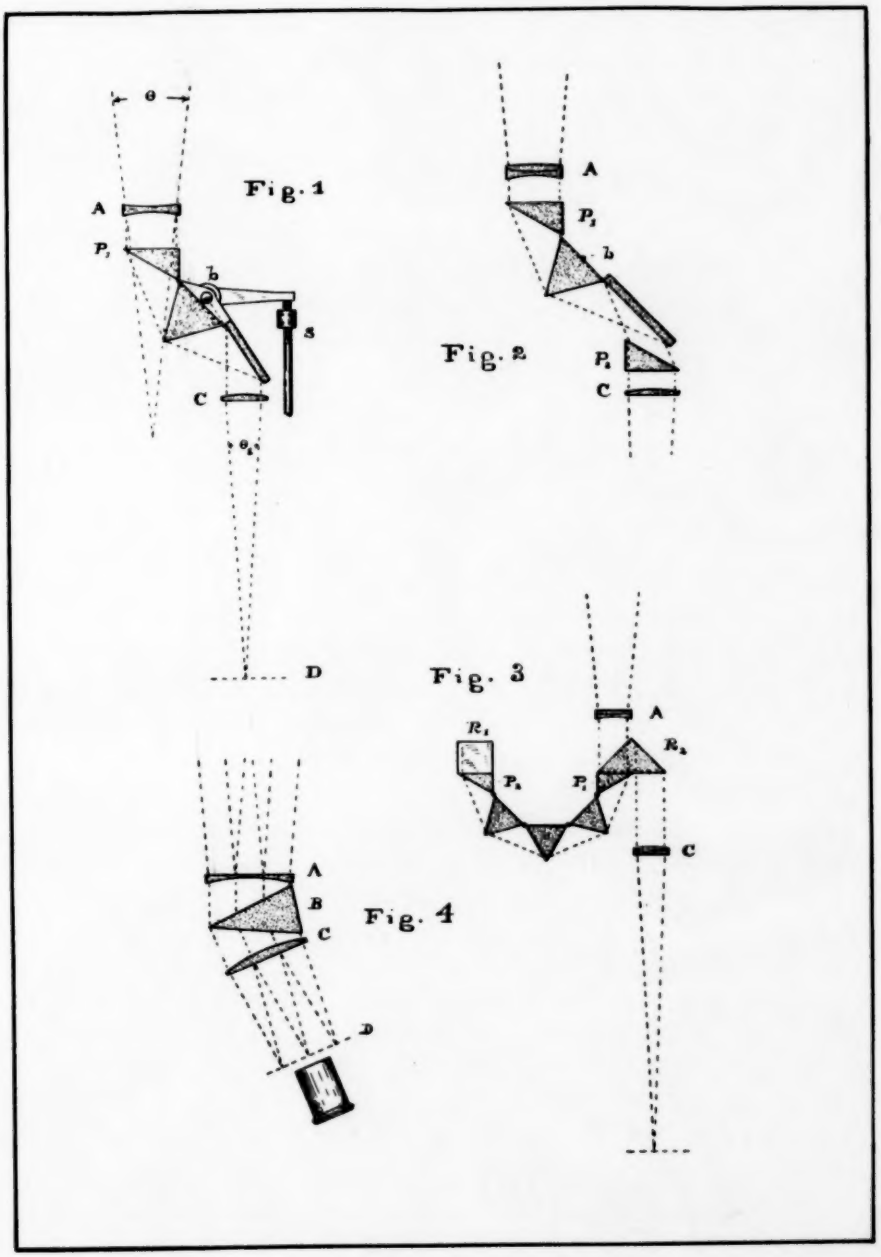
$$r_1 = 845\,\text{mm (33.25 in.)} , \qquad r_2 = 93.8\,\text{mm (3.695 in.)} .$$

The arrangement of the parts of this instrument with reference to each other and to the large speculum is shown diagrammatically in Fig. 4, Plate I.

It is frequently very necessary or desirable that the angular aperture of the positive image-forming lens should be much smaller than the large objective and negative lens. Thus, to obtain full photographic resolution on a photographic plate placed at D, Figs. 1 and 4, the focal length of the lens C should be from twenty-five to forty times its aperture.[1] In such cases it is impossible to compensate the aberration of the first lens in the manner adopted in the case of the instrument last described, and, if large linear apertures of the collimator are to be used in conjunction with the main objective, it becomes necessary to correct the aberration of the negative collimator independently

[1] "Further Notes on Astronomical Spectroscopes," ASTROPHYSICAL JOURNAL 3, 187 *et seq.*, March 1896; "Conditions of Maximum Efficiency in the Use of the Spectrograph," ASTROPHYSICAL JOURNAL 3, 327 *et seq.*, May 1896.

PLATE II.



of the second positive lens. This may be done, as already pointed out, by the use of a compound lens of glass. For work in the ultra-violet where glass (especially dense flint) is objectionable on account of absorption, we may construct the compound lens of quartz and fluorite. Or better still, especially in the case of a reflecting telescope, we may use in place of a lens a convex mirror, as in the Cassegrainian type of instrument, the small mirror being in this case parabolic (not hyperbolic) in cross-section and placed at its own focal length from the focus of the large objective. The prism train and image-forming train may then be placed at any point in the parallel beam of light reflected from the small mirror; most conveniently, perhaps, just behind the large objective, as shown in diagram in Fig. 5, Plate I. The reflecting collimator may also be used where the large objective is a refractor in such a way as illustrated in Fig. 6 (same plate). In such cases, however, it is not so convenient as the negative lens, and the latter, moreover, has the added advantage already pointed out that we may by its use correct also for the chromatic aberration of the large objective.

It is obvious that any desired form or arrangement of prism train may be used with these instruments as readily and easily as with the compound slit spectroscope. A number of other trains of different form and arrangement of prisms which seem well adapted to this type of instrument are shown in the various figures of Plate II. Other modifications of these or of the usual simple prism trains will readily suggest themselves. In Fig. 1 of this plate the dispersing system consists of a fixed arm combination of type 4 described several years ago by the writer,[1] with an added half-prism P, which secures about 50 per cent. added resolving power and at the same time reduces the horizontal width of the emergent pencil of rays, and consequently reduces the horizontal angular aperture θ_1 of the image-forming lens C, thus necessitating a smaller focal length of the latter to obtain full photographic resolving power at D. If the latter

[1] "Fixed Arm Spectroscopes," *Phil. Mag.*, **38**, 348, October 1894; also *A. and A.*, December 1894.

advantage is unimportant we may add still more to the resolving power and secure symmetry of train by placing a second half-prism, P_2, in front of the lens C as shown in Fig. 2. It is obvious that, when these half-prisms are placed as shown in these figures, with their short sides perpendicular to the axes of the respective lenses A and C, they themselves require no adjustment for minimum deviation, and do not interfere in any way with the performance of the fixed-arm system B, by means of which any desired part of the spectrum can be brought to the center of the field of view by simply rotating the prism mirror system about the axis b by means of the screw s or any equivalent device. Such half-prisms therefore might be used with advantage in other cases where the "fixed-arm" systems have been employed.[3]

In Fig. 3 is shown another type of "fixed-arm" dispersing train which is particularly adapted to those lines of work in which it is necessary to change the resolving power of the instrument without disturbing the position of the collimator or eyepiece.

The beam from the negative collimator A falls first on the lower part of a half-prism P_1, thence passes through the train B to a second half-prism P_2 at the end, thence is returned by a double-reflection right-angled prism R_1 through the upper half of the prisms P_2, B, and P_1. On emergence from the upper half of P_1 it is caught and returned to the image-forming lens C by a second double-reflection right-angled prism R_2 placed with its axis at right angles to the axis of R_1. Any desired dispersion and resolving power down to one (two half) prism can be secured by changing the number of prisms in the intermediate train B.

This new form of focal plane spectroscope would seem to be particularly well adapted to certain lines of astronomical investigation which require a large instrument, either refractor or reflector, and which cannot well be undertaken with the com-

[3] *Loc. cit.;* also ASTROPHYSICAL JOURNAL 1, 232, March 1895; *ibid.*, 2, 264, November 1895; *ibid.*, 3, 169, March 1896; etc.

pound slit spectroscope, because of its nature, nor with the objective spectroscope, because of its cost. Among these may be mentioned:

1. The photographic survey of the spectral type of very faint stars. This is a field of work which, as Professor Pickering has demonstrated, can be carried on most rapidly and satisfactorily with the objective prism. But in the study of very faint stars very large apertures and consequently very large prisms are necessary. The expense of the latter may be avoided by using in place of the objective prism one of the forms of focal plane spectroscope above described. In order to secure the spectra of a number of stars at once, as is possible with the objective spectroscope, the aperture of the spectroscope should be made considerably larger than that required to receive the cone of rays from the large objective. In this case, in order to avoid the effects of aberration and astigmatism in the lateral images and secure as large as a field of good definition as possible, the two lenses are made of the same material and focal length and are each concavo-convex in form as shown in Fig. 4, Plate II.

2. Photographic study of the spectra of faint individual stars, particularly of the ultra-violet region. This is the field of work to which Professor Keeler proposed to apply the instrument which has already been described. It is easy to obtain quartz prisms, lenses, etc., of the size required for the focal plane instrument, while it would be quite impossible to obtain them of a size required for an objective spectroscope.

3. Photometric studies of the relative spectral intensity of faint stars and variables. In this field again, as well as in the others, the focal plane instrument has the advantage over the objective prism in that the spectroscopic resolving power can be readily and inexpensively varied to suit the requirements of the investigation in hand by the use of a form of instrument similar to that shown in Fig. 3, Plate II, and already described.

The instruments are applicable to many other lines of work which it is unnecessary to mention in detail. In the improved

form they will occupy, I think, a much more important field of usefulness than has been filled in the past by the ocular spectroscope.

In addition to the two instruments for the Lick and the Leander McCormick Observatories described in this paper, a third and much larger one, of 70 mm or 2¾-inch aperture, has been designed for use with the 13-inch (33 cm), 18-inch (46 cm), and 30-inch (76 cm) telescopes of the new Allegheny Observatory[1] which is now approaching completion. This last instrument is particularly designed for use in the fields of work (1) and (3) above described.

ALLEGHENY OBSERVATORY,
December 1900.

[1] See "Report of Director for Year Ending December, 1900," *Misc. Scientific Papers of the A. O., No. 1*, Murdoch Kerr Press, Pittsburg.

THE ARC SPECTRUM WITH HEAVY CURRENTS.

By W. B. Huff.

At the suggestion of Professor Ames, the writer undertook a study of the effects produced on the arc spectrum of various substances by the use of heavy currents. The arrangement of resistances was such that the 110-volt direct current used could be varied from two amperes, the smallest for which the discharge would pass, to over two hundred amperes. The exposures for the two extreme currents in a given case were made on the same plate, all possible precautions being taken to prevent accidental disturbances of the camera.

Metals of low fusing points—such as lead, zinc, and cadmium—when placed in carbon holders in such quantities as practically to constitute one pole of the arc, showed fairly definite absorption spectra, which became sharper with heavier currents and longer exposure. A comparison of the plates obtained by using extreme values of the currents gave no definite results.

The spectra of more refractory substances, such as calcium, were brought out very strongly by a current of one hundred amperes, though the relative intensities were about the same as for a current of but a few amperes.

When a large amount of calcium was used, the H and K lines were reversed so sharply that it was possible, by using much less of the metal, to obtain these same lines, fine and clear, in the reversals. These finer lines were not symmetrically placed as to the reversals, illustrating the "density-shift" noted by various observers.

Because of the importance of the iron spectrum in astrophysical work, it was deemed advisable to obtain some plates of this spectrum in the region of the violet. The extreme values of the currents were two amperes and about two hundred and fifty amperes. The times of exposure were about three

seconds for the strong currents and ten to twenty minutes for the weak ones. But the discharge with heavy currents took place in a series of sharp explosions which usually blew out the arc and made time estimation difficult.

As would be expected, the spectrum of such a violent discharge showed a good deal of continuity, as well as many reversals and shaded lines. The results from these two extreme currents were compared in order to detect possible shifts, and for change of relative intensities. Though the sharpest plates were chosen for measurement, the character of the heavy-current spectrum made accurate settings difficult. Various pairs of lines of the two spectra were found slightly displaced with respect to each other, but the shift was not uniform, and in no case was it more than a few thousandths of a unit. Errors of measurement and the probability of the lines being shifted differently, owing to differences of pressure and of density at different times of exposure, as well as at different parts of the arc, make it impossible to say anything more definite. The relative intensities of the two plates were not greatly different, though in neither case did they agree with those assigned by other observers; *e. g.*, with those of Kayser and Runge.

The most interesting difference in the spectra of the arcs from these extreme currents was in the case of the bands appearing in the spectrum of ordinary carbon poles, one group of which is generally ascribed to carbon or one of its oxides; the other, to cyanogen. The different bands of the "cyanogen" group seemed to come out with about the same relative intensities among themselves, and a similar remark applies to the "carbon" group. But the two groups come out very differently in arcs from small and from large currents. Many plates were taken, and all showed that, as the current was increased, the carbon bands came out more and more strongly as compared to those of cyanogen. The latter came out clearly for very small currents, even when those of carbon are scarcely to be detected. Of course the cyanogen bands come out more strongly with increasing currents, but their increase of intensity by no means

equals that of the carbon bands for the same increase of current. The extreme currents used were two amperes and one hundred and eighty amperes.

In order to investigate more closely the structure of the arc, it was studied objectively with the flat grating, in the manner suggested to me by Mr. L. E. Jewell. For this kind of work the plane grating is obviously superior to the concave, avoiding, as it does, the integrating effect of the latter.

Studying the arc as a whole, the observations of previous observers[1] were confirmed in many particulars. Using the arc between ordinary commercial carbons, the lines due to metallic impurities are seen only in the immediate neighborhood of the negative pole; the bands of carbon and of cyanogen appear to have their origin at the positive or hot pole. If larger amounts of metal are put into either pole of the arc, the metallic lines may extend across to the positive pole. But in no case was a short metallic line observed originating at the positive pole. Increasing current also causes these lines, which appear as short eruptions of metallic vapor, to extend entirely to the hot pole. With sudden changes of current, these prominences lengthen or shorten, but so slowly that the change in length is readily followed with the eye. The heads of the carbon bands can be traced as extremely brilliant lines on the continuous spectrum coming from the hot pole. The cyanogen bands were not traced so far. It seems possible, therefore, that the radiation giving rise to the cyanogen group is to be thought of as coming from the surface of the arc discharge, although showing up more brilliantly nearer the hot pole. The ordinary quiet discharge usually takes the well-known curved path. If the poles are brought closer together, the discharge changes into the hissing form and appears to take the shortest path between the terminals. As would be expected from this shortening of the path, the resistance of the hissing arc is less than that of the quiet one. For a given separation of the poles, the discharge may be unstable, taking first one form, then the other. But even then the discharge with the shortest path has the least resistance.

[1] BALDWIN, *Phys. Rev.*, 3, 1895; FOLEY, *ibid.*, 5, 1897; THOMAS, *C. R.*, 119, 1894.

For the hissing arc there is considerable continuous spectrum, and the carbon bands flash out strongly at the hot pole; more strongly, indeed, than the same current shows when it is passing quietly. If a blast of air is directed against the silent arc in such a direction as to shorten the path of the discharge, the arc assumes the hissing form and the resistance may be decreased as much as 20 per cent. As a result of the increased current, there is an increase in the intensity of the carbon bands in the hissing arc. But, in the arc subjected to the air blast, the cyanogen bands seem at times to come out more strongly. This goes to support the idea that the cyanogen bands are a surface effect in the arc. When the arc is studied objectively, it is possible to get the carbon bands much more sharply defined than those of cyanogen. It is as if the latter came from the outer sheath of the arc.

If an alternating discharge is used, both groups of bands extend entirely across the space between the poles, and appear to be equally strong at the two poles. The prominences, due to a metal which is present in the arc in considerable quantity, also extend across this space. Analysis with a revolving mirror would probably show that such lines are shot out from the poles alternately.

The analogy between this discharge of particles proceeding from the negative pole to the intensely heated positive pole, and the kathode discharge heating the anti-kathode of a vacuum tube, is at least worth noting. Both the arc discharge and that in the vacuum tube are deflected by a magnet, though Röntgen rays have not been obtained from the arc.

In conclusion, then, it may be said that heavy currents bring out the carbon bands much more strongly than they do those of cyanogen; and that, in an arc between carbon poles, small amounts of metal show metallic lines only at the cool pole, while the bands of carbon and cyanogen appear strongest near the hot or positive pole.

PHYSICAL LABORATORY,
JOHNS HOPKINS UNIVERSITY,
May 1902.

THE SPECTRA OF CATHODO-LUMINESCENT METALLIC VAPORS.

By PERCIVAL LEWIS.

RECENT investigations have in general tended to confirm the view advanced by Hittorf[1] and elaborated by E. Wiedemann,[2] that the luminosity of gases and vapors, particularly in the case of vacuum-tube phenomena, is of the nature of "phosphorescence;" that is to say, it is directly dependent, not upon high temperature, but upon chemical, electrical, or unknown processes. All such phenomena of radiation, to which Kirchhoff's law does not quantitatively apply, may, until our knowledge permits of further differentiation, be grouped under the general title "luminescence," suggested by E. Wiedemann.

Since fluorescence is one of the most striking luminescent phenomena, it was of interest to discover whether substances possessing a finite number of definite free periods of vibration, such as gases and vapors, could be made to fluoresce. This question was answered affirmatively by E. Wiedemann and G. Schmidt,[3] who showed that sodium and potassium vapors fluoresce under the action of sunlight, giving characteristic spectra, in part corresponding to the ordinary flame spectra.

Cathode rays are far more effective than sunlight in producing fluorescence of solids, and it seems reasonable to expect similar effects in the case of vapors, independent of the direct effect of the current passing through them. Some instances substantiating this expectation are well known. The negative glow from a plane cathode extends not only toward the anode, or in the direction of the current, but from the opposite side of the cathode as well, even when all precautions are taken to

[1] HITTORF, *Pogg. Ann.*, **7**, 587, 1879.

[2] E. WIEDEMANN, *Phil. Mag.*, **28**, 149, 248, 1889; *Wied. Ann.*, **37**, 177, 1889.

[3] E. WIEDEMANN and G. SCHMIDT, ASTROPHYSICAL JOURNAL, **3**, 207, 1896; *Wied. Ann.*, **57**, 447, 1896.

avoid a flow of current in that direction; the length of this negative column increases as the pressure diminishes, and when it reaches the walls of the tube they begin to fluoresce. Hertz,[1] in 1883, isolated the effects of the cathode rays from those of the current by constructing a vacuum tube in which the anode was concentric with the cathode, and almost in the same plane, so that the stream lines were limited to a small space, while the cathode rays were projected to the end of the tube, 30 cm distant. Mercury vapor at the end of the tube glowed under the action of the cathode rays, and showed the characteristic strong lines of the mercury spectrum. Lenard[2] found that cathode rays would pass through a thin aluminum window, into gases at atmospheric pressure; these gases glowed, but too faintly to show any spectral lines. In this case also electric currents could have played only a very small part, as they could have arisen only from small static charges.

Some investigations on the vacuum tube spectra of metals have been published,[3] but, with the exception of the one experiment by Hertz, no attempt seems to have been made to isolate the cathodo-luminescence from the direct effects of the current. The writer has investigated the effects of cathode rays upon the vapors of such volatile metals as were available, and has observed a characteristic luminosity in a number of cases. With sodium and potassium these effects seemed somewhat different from those observed by Wiedemann and Schmidt when these vapors were fluorescing under the action of sunlight. The cathode rays may merely produce a stronger fluorescence, or the effects may be excited by the mechanical impact of the cathode corpuscles, or by their electrical charges, in a manner altogether different from fluorescence, so it may be safer to apply the more general term "luminescence" to the observed phenomena. It seems evident, however, that this radiation can depend neither upon

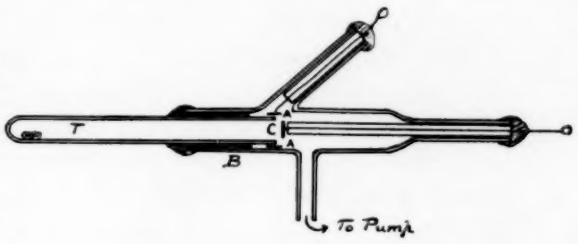
[1] H. HERTZ, *Wied. Ann.*, **19**, 809, 1883.

[2] P. LENARD, *Wied. Ann.*, **51**, 229, 1894.

[3] See, for example, E. WIEDEMANN and G. C. SCHMIDT, *Wied. Ann.*, **57**, 454, 1896; A. C. JONES, *Wied. Ann.*, **62**, 30, 1897.

high temperature nor upon an electric current in the ordinary sense, and we might expect to see only the more fundamental spectral lines.

The vacuum tube used was somewhat like that used by Hertz, and is shown in the accompanying diagram. The substance to be investigated is placed in the hard glass tube *T*, which is then telescoped into the tube *B*, and joined to the latter with sealing wax. From the disk-shaped cathode *C* the cathode rays are projected to the bottom of the tube *T*, at a distance of 25 cm. The ring-shaped anode *A* lies outside of *T*, at a distance of about 5 mm from *C*, and almost in the same plane. This arrangement gives a compact system of current lines, and shields the bottom of *T* from cathode rays when the current is reversed. As a further precaution against stray currents and electrostatic effects, a wire gauze cylinder or a long wire loop was sometimes inserted throughout the entire length of *T*, without sensibly altering the observed phenomena. The tube was exhausted with a Töpler-Hagen pump, and excited with a coil giving a spark about 15 cm in length. The luminous vapors were examined with a Browning pocket spectroscope or a chemical spectroscope with calibrated scale for identifying the lines. In the case of some lines too weak to see with the latter instrument, a rough identification was made by comparison with the position of the hydrocarbon bands in the spectrum of the Bunsen flame used to heat the tube. These estimates are of course liable to error, but such lines probably correspond to known strong lines of the metal.



Below is a brief statement of the results obtained. None of the metals used were chemically pure. The atmosphere within the tube was chemically prepared nitrogen.

Sodium. — At a vapor pressure corresponding to a temperature below red heat, an orange-colored glow was observed, showing the D lines; at red heat the glow became greenish-yellow, and the citron-green lines λ5683–88 appeared; also faint lines or bands in the red and blue-green, probably the pairs λ6154–61 and λ4979–83. This luminescence and that in the other cases described, appeared only when the exhaustion had reached such a point that the cathode rays struck the bottom of *T* and produced fluorescence of the glass. It disappeared when the cathode rays were deflected with a magnet, but was not affected by the introduction of a gauze cylinder or long wire. It did not appear when the current was reversed or when, at pressures slightly above cathode-ray pressure, a small conduction current was transmitted through the vapor by touching the end of *T* with an earthed conductor. These facts indicate that the observed effects were due to the cathode rays alone. The color and spectrum of the luminescence were different from those of the fluorescence seen by Wiedemann and Schmidt, which was green, and showed green and red flutings.

Potassium. — A light purple glow. The yellow sodium lines were the brightest in the spectrum. In addition, there were visible the yellow potassium lines λ5783, 5802, and 5832, and several faint lines in the green, not bright enough to be identified. The red and violet lines were not visible, which is not surprising, considering how near they lie to the limits of the spectrum, but the color of the luminescence left little doubt that their radiations were present.

Magnesium. — Magnesium powder was heated in the tube. It did not melt, but decrepitated actively and sublimed. At dull red heat a bright green glow filled the end of the tube, showing the triplet λ5183, 5172, 5167.

Mercury. — The glow was pale green, not pink, as described by Hertz. A difference of atmosphere may account for this. The yellow, green, and blue lines were observed, the green line being strongest and most persistent.

Zinc. — Just below red heat, a faint lilac glow was seen, giv-

ing the triplet λ4811, 4722, and 4680. These lines are sometimes seen in the flame spectrum of the chloride. At red heat a weak red line (probably 6383) also appeared, and the glow changed to a rich purple. These lines are the only ones which appeared in a feeble spark spectrum, without condenser, and are the longest lines of the spark spectrum. Sometimes λ4912 was faintly seen with the pocket spectroscope.

Cadmium. — Below red heat the glow was pale lavender; at red heat, violet. The lines observed, in the order of their intensity, were λ5086, 4413, 4800, and 4678; at very high temperatures a red line also appeared (probably 6431). The lines 5086, 4800, and 4678 can be seen in flame spectra of the chloride. It is remarkable that 4413, which is almost invisible with a simple spark, and relatively weak with small condenser, was next to the green line in intensity, while 4678 was very weak. A. C. Jones[1] states that in the vacuum tube spectrum of cadmium the color is green without a spark-gap, blue with it, and that, without a spark-gap, λ4800 and 4678 are strong, and 4413 weak. The longest lines in the spark spectrum are λ5086, 4800, and 4678. These differences in color and spectrum are additional evidence that the effects observed here are essentially different from those attending the passage of the current through the vapor.

Thallium. — At comparatively low temperatures a bright green luminescence was produced, which showed the green line λ5380.

Bismuth, lead, antimony, tin, and aluminum showed no appreciable luminescence.

Sulphur, selenium, and tellurium gave an almost inappreciable blue glow, which showed a very weak spectrum, apparently continuous in the green and blue.

In most cases, the lines observed were those which are seen in flame or weak spark spectra, and seem, therefore, to correspond to fundamental types of vibration.

Further investigation of this subject will be carried on.

UNIVERSITY OF CALIFORNIA,
Berkeley, May 1902.

[1] A. C. JONES, *Wied. Ann.*, **62**, 30, 1897.

A REPLY TO THE RECENT ARTICLE BY LOUIS BELL.

By A. PEROT and C. FABRY.

IN an article recently published in this JOURNAL[1] Dr. Louis Bell criticises our interference methods for the measurement of wave-lengths, and in particular our work on the corrections to Rowland's scale. His conclusions may be summed up as follows: (1) Interference methods do not give results which are more precise than those obtained with gratings; (2) our correction curve to Rowland's tables seems to be based on inadequate evidence. We desire to reply briefly to these criticisms.

So far as absolute values (comparison with the meter) are concerned, we have naturally taken as the point of departure the result of Michelson and Benoit, which is unquestionably the most precise. Reference to old values is only of historical interest. We are, therefore, concerned only with comparisons, or relative measures, which are much easier and more precise than absolute measures. Thus the quantity one-millionth, given as the probable error of the results obtained by Michelson and Benoit, applies to the absolute values; the relative wave-lengths are much more precise, as we have frequently had occasion to verify for the green and red lines.

In two places in his article Dr. Bell insists upon the discordance between the values found by Michelson and by Hamy for the relative wave-lengths of the green and red lines of cadmium. But it must not be forgotten that two different sources were employed (Hamy used the discharge of a condenser through a tube without electrodes); the wave-lengths are probably different, and although this point calls for further researches, it is unfair to attribute the differences to the interference methods. It is none the less improper to condemn the green cadmium line as a standard of wave-length, as Dr. Bell seems to do. As a matter of fact, the use of the lines of metallic vapors at low pressures,

[1] ASTROPHYSICAL JOURNAL, **15**, 157, April 1902.

as standards, constitutes a marked advance in spectroscopy. During an experience of five years in spectroscopic and metrological measurements, we have never observed an error which could be attributed to a variation of the cadmium lines employed as a standard. The discovery of a brighter luminous source is certainly to be desired, but it is difficult to conceive of one giving better defined wave-lengths.

All comparisons between the values found in the arc and in the solar spectrum seem to us incompetent to solve the problem of the precise comparison of wave-lengths. Mr. Jewell's direct comparisons show the displacements, usually in the direction of an increase of wave-length for the solar spectrum, but occasionally in the opposite direction. It is certain that these variations demand further investigation, but it does not seem reasonable to endeavor to solve a simple and clearly defined problem through another which is much more complex.

There is no occasion for surprise in the following fact, alluded to by Dr. Bell: If we start with the values found by us for the lines of *Zn* or *Na*, and apply our corrections to reduce them to Rowland's scale, values about 0.02 tenth-meters smaller than Rowland's are obtained. Rowland's values are relative to the solar spectrum, while ours refer to artificial sources at low pressure (a vacuum for *Zn* and atmospheric pressure for *Na*). The differences between Rowland's values and our own is in the direction and of the order that one would expect.

Finally, if our corrections to Rowland's scale were due to ordinary experimental errors, the points of our diagram would be irregularly distributed on both sides of a horizontal line; that this is not the case may be seen at a glance. It appears incontestable that, if two lines are selected in Rowland's tables, one near λ 5300 and the other near λ 5600, the ratio of the two values given by Rowland is in error by about seven thousandths; in other words, if one were correct, the other would be in error by about 0.03 or 0.04 tenth-meter. This can in no wise diminish our admiration for the immortal work of Rowland, but the importance of this work should be a further reason for seeking to derive from it the best possible results.

NOTE ON THE NEBULA SURROUNDING *NOVA PERSEI.*

By Louis Bell.

The photographs by Ritchey, and the notes by Perrine, in the April Astrophysical Journal seem to afford a new basis for the consideration of the apparent very rapid motion in the nebular appendages of the *Nova*. The chief nebular ring in Ritchey's photograph of September 20, having a radius of about 12′ 30″, indicates, if its expansion rate be taken as that of electromagnetic waves for the 210 days following the outburst, a parallax of 0″.02, and a distance of 159 light years. The contour of the ring suggests that it lies nearly in the normal plane. If it does not so lie, then the radius observed shows either a propagation-speed greater than V, or a parallax greater than 0″.02.

Perrine's photograph of March 29 previous gives the principal ring a radius of about 2′, which also leads to a parallax of almost exactly 0″.02 on the same assumptions as before. Hence, if the one ring is an expansion of the other from whatever cause, there is no evidence of acceleration, and any hypothesis of projected matter of ionic or other dimensions gaining velocity under a repulsive force, becomes untenable. The case is no better for projected matter, if the velocity is assumed to be due to an initial explosion, since, aside from the magnitude of the forces involved, matter cannot be driven with or without constant acceleration to a velocity V by a force having a propagation-speed less than V under such circumstances.

The expansion of the inner rings on the dates mentioned is evidently not in the normal plane, but unites in showing non-accelerated motion. In the September photograph there is a nebulous patch 20′ from the *Nova*, which may or may not be part of the system. If it is, then the parallax would be nearly 0″.03, or the expansion speed proportionately greater. Now

Chase, Aitken, and others unite in a very small value of the parallax of the *Nova*, little, if at all, larger than 0".01. Hence V is certainly as small a propagation-speed as agrees with the facts, and there is at least a possibility of having to account for a larger value. Of propagation-speeds greater than V, but two are known even hypothetically: that of change of gravitational potential, and that of a compressional wave in the ether. Of the former nothing is known; of the latter, only that it would equal $\sqrt{\frac{k+\frac{4}{3}n}{\rho}}$. As we have only inferential knowledge of the bulk-modulus of the ether, speculation seems uncalled for, until the parallax is finally shown to be below 0".02.

Of hypotheses to account for the propagation of nebulosity outward with velocity V, the most obvious is that of Kapteyn, independently devised by Seeliger and by one or two others. A modification of this fundamental idea based on the secondary effects of an electro-magnetic wave-front was suggested by the writer to Professor Hale in December last; and, as recent data have some bearing on the matter, it may be worth a brief discussion. There are three rather serious objections to the hypothesis of pure reflection. First, reflected light, whether reflected in the ordinary way from heterogeneous surfaces or from small particles, would be polarized, and Perrine's report on this feature of the case indicates absence of polarization. Second, reflection does not adequately explain the very remarkable persistence of some regions of strong nebulosity at a small angular distance from the *Nova*. Especially the nebular peak nearly south of the *Nova* has an intensity all out of proportion to that of the outer ring, while both on the reflection hypothesis should be at similar radial distances. If they are, then the ring must represent a condition of matter having a very small albedo compared with that in the other region. Third, at the radius of 210 light days denoted by the radius of the ring of September 20, reflection does not adequately account for the brightness of the nebular matter observed.

Taking the *Nova* as of zero magnitude at its maximum, and of

surface brilliancy equal to the Sun, it would have from the computed parallax about 2500 times the total solar light. A material body at $R = 36{,}400$ would receive from such a source a light a little stronger than full moonlight on the the Earth, and at even the high albedo of *Mars* would reflect light about as intense as $\frac{1}{2 \times 10^6}$ of terrestrial sunlight. Since the distance implied by a parallax of 0".02 is about 10^7 units, even a solid body of the dimensions of the *Nova* itself would be far below any magnitude within reach of existing instruments; and it is extremely doubtful whether even a matter-charged region of the extent of the nebular wisps observed would, at any admissible value of the mean albedo, be within the present reach of photography, and still less give images of the density found.

If, however, one considers the luminous disturbance as a secondary result of radiated electro-magnetic strains, somewhat similar in character to those observed in solar storms, the case is more promising. Every electro-magnetic wave-front carries an orthogonal system of electric stresses. When of sufficient magnitude, these are capable of exciting powerful luminosity in masses of tenuous gas. Air at a pressure *circa* 0.01 mm becomes a rather good conductor — as good as the best electrolytes — and currents set up in such a medium give spectra of the gases involved very much modified from their usual forms, and nearly or quite free from the lines due to any intermingled metallic particles.

In comparatively trivial solar storms the envelope of tenuous gas 100 km or so above the Earth becomes the seat of powerful disturbances of this sort manifest as widespread auroras and as the source of violent changes in the magnetic elements accompanied by great local variations in Earth potential.

Admitting that a solar disturbance involving, say, $\frac{1}{500000}$ of the visible solar area is capable of such effects, then a disturbance of similar intensity involving the whole surface of a body of the magnitude of the *Nova* would set up, even at $R = 36{,}400$, electric forces about equal to those known to act on the Earth and its gaseous envelope.

Matter-charged space filled with tenuous gas dissipated from the matter is the necessary seat of currents and the transformation of energy when subjected to such stresses. Such is the normal result in the upper atmosphere, although the luminous effects are of only moderate brilliancy, owing to the small thickness of the strata in which the action takes place. In cosmic masses there may well be very brilliant results. For example, even so small a volume as a cube 1000 km on the edge, subject to a potential gradient of 10 volts per km, would transform an amount of energy sufficient to give the mass a surface brilliancy not less than 50 times as great as that which could be due to pure reflection at any reasonable value of the albedo. The effect may be compared to that of an "end-on" Geissler tube of colossal dimensions. The light, moreover, would be unpolarized, and would give a relatively simple gaseous spectrum of which the lines would not necessarily be identical in number or relative intensity, even were the gases in the spaces affected of similar tenuity and composition. These discharges in spaces not possessing definite electrodes are quite freaky in the matter of spectrum produced.

This view of the case would strongly support Seeliger's hypothesis, since it requires a similar distribution of matter, not in dust, which would produce relative opacity, but in masses of moderate dimensions. According to it, any considerable electric disturbance in matter-charged space would produce luminosity, which should then be an ordinary concomitant of the aggregation or disintegration of masses in or near such space. It permits relatively rapid changes of nebular luminosity, and considerable variations in the spectra in various parts of a single nebula.

If, in the case of the *Nova*, one is dealing with three superposed spectra—viz., a continuous spectrum with absorption bands, due to the main body concerned; a true gaseous spectrum from surrounding incandescent gases, and this secondary gaseous spectrum—some of the changes observed become more comprehensible. Moreover, masses not gathered up by the principal

body, and carrying off their disturbing action in hyperbolic orbits through the nebular region, are competent to explain the persistence of luminosity in the line of the *Nova's* path. If there were present near the *Nova* a sufficiently retarding envelope of matter to convert these orbits into ellipses for any considerable swarm, the somewhat irregular light-variations observed become explicable, and must be greatly modified or disappear as the primary disturbances and the resulting secondary ones here noted die away. This electro-magnetic hypothesis in no way controverts the existence of cosmic nebular masses at the temperature of true incandescence, but it does avoid the necessity of such an explanation for every tenuous nebular wisp that gives a bright-line spectrum, and it does represent a condition which is not unknown in the laboratory.

THE SPECTRA OF POTASSIUM, RUBIDIUM, AND CÆSIUM, AND THEIR MUTUAL RELATIONS.[1]

By HUGH RAMAGE.

THE spectra of this group of metals have already been considered in a paper by the author on "A Comparative Study of Spectra," etc.[2] It was there shown that the differences between the principal series of lines in these metals depended on the atomic mass alone; and also that there was a close connection between the subordinate series and the atomic mass. A further study of the latter series was impossible at the time of writing the paper, owing to the fewness of the lines which had been observed and measured in them; practically no lines were known in the second subordinate series of rubidium and cæsium.

Some lines belonging to the subordinate series have been measured in Bunsen-flame and spark spectra by Lecoq de Boisbaudran, and in the arc spectrum by Liveing and Dewar, and by Kayser and Runge. Lehmann has measured some lines in the arc spectra in the red region. Lines recorded by these observers were found by the writer, with considerable intensity, in the oxyhydrogen flame spectra of the metals; and other lines, weaker than the above, were present which had never been recorded. Photographs of these high-temperature flame spectra were taken with a spectrometer designed by Professor Liveing, fitted with a Rowland plane grating ruled with 14,438 lines to the inch. The quartz lenses were plano-convex, with a focal length for the D lines of about 778 mm. The spectra in the first and second orders were photographed, and some measurements were made in the red region by eye observations. Spark spectra were photographed, superimposed on the flame spectra, of iron and titanium principally, but other metals were also employed.

[1] From advance proofs, sent by the author, of a paper read before the Royal Society, on June 5, 1902.

[2] *Roy. Soc. Proc.*, **70**, 1, 1902.

These furnished the numerous fiducial lines required for the accurate determination of the wave-lengths.

The lines in the subordinate series are generally more diffuse

CÆSIUM.

Wave-length	Oscillation frequency, in vacuum,	Intensity	Previous measurement	Observer	Series and number
6984	14314	6			
74	335	9	6973.9	K. and R.	I_4
6869	554	2			
29	639	2			
6722	873	9	6723.6	"	I_4
6630	15079	2			
6590	171	8	.....	.....	II_4
6472	447	2			
6433	540	2			
6354	733	8	.....	.....	II_4
6217.6	16078.7	2			
13.33	89.7	8	6213.4	K. and R.	I_5
6034.43	16566.7	4	.....	.....	II_5
10.59	16632.4	8	6010.6	K. and R.	I_5
5847.86	17095.6	2			
45.31	102.7	8	5845.1	"	I_6
39.33	120.2	2	.....	.....	II_5
5746.37	397.2	1	.....	.....	II_6
5664.14	649.7	7	5664.0	K. and R.	I_6
35.44	739.6	5	35.1	"	I_7
5574.4	933.9	1	5572	L. de B.	II_7
68.9	951.6	1	.....	.....	II_6
03.1	18166.2	3	5501.9?	K. and R.	I_8
5466.1	289.2	4	5465.8	"	I_7
14.4	463.8	1	.....	.....	I_9
07.5	487.3	1	.....	.....	II_7
5351	682	1	.....	.....	I_{10}
5341.15	717.0	3	5345	L. de B.	I_8
5304	848	<1	.....	.....	I_{11}
5256.96	19016.8	1	5257	L. de B.	I_9
5209	192	<1			
5199	228	<1	.....	.....	I_{10}
5154	396	<1	.....	.....	I_{11}
4593.30	21764.8	8	4593.34	K. and R.	P_2
55.46	945.6	10	55.44	"	P_2
3888.75	25707.9	2	3888.83	"	P_3
76.31	790.4	4	76.73	"	P_3
3617.49	27635.7	<1	3617.08	"	P_4
11.70	680.0	2	11.84	"	P_4
3477.25	28750.3	1	.....	.....	P_5
3398.40	29417.3	1	.....	.....	P_6
48.72	29853.7	<1	.....	.....	P_7
3314	30166	<1	.....	.....	P_8
3287	30414	<1	.....	.....	P_9

than those in the principal series. Some of the weaker lines, notably those of cæsium, are very broad with diffuse edges; very accurate measurement of these is impossible.

Particulars of the spectra are recorded above; the oscillation frequencies are reduced to their values in a vacuum. The lines

RUBIDIUM.

Wave-length	Oscillation frequency, in vacuum	Intensity	Previous measurement	Observer	Series and number
			7950.46	Lehmann	P_1
7799	.....	Very strong	7805.98	"	P_1
6306.8	15851.3	1			
6299.19	870.5	9	6297	L. de B.	I_4
6206.74	16106.8	8	6203	"	I_4
6160.04	228.9	5	6159	"	II_4
6071.04	466.8	4	.....	.....	II_4
5724.62	17463.2	8	5724.41	K. and R.	I_5
5654.16	680.9	3	5654.22	"	II_5
48.19	699.6	7	48.18	"	I_5
5579.3	918.1	2	.....	.....	II_5
5432.05	18403.9	6	5431.83	K. and R.	I_6
5391.3	543.0	1	.....	.....	II_6
63.15	640.3	5	5362.94	K. and R.	I_6
22.83	781.5	1	.....	.....	II_6
5260.51	19004.0	4	5259.8	K. and R.	I_7
34.6	098	1	.....	.....	II_7
5195.76	240.7	3	5194.8	K. and R.	I_7
65.35	354.1	2			
51.20	407.2	2	.....	.....	I_8
5132	480	<1	.....	.....	II_8
5089.5	642.5	1	.....	.....	I_8
76.3	693.6	1	.....	.....	I_9
37	847	1			
23	902	1	5021.8	K. and R.	I_{10}
17	926	<1	.....	.....	I_9
4983	20062	<1	.....	.....	I_{11}
67	127	<1			
4215.68	23714.4	9	4215.72	K. and R.	P_2
02.04	791.4	10	01.98	"	P_2
3591.86	27832.8	3	3591.74	"	P_3
87.27	868.4	4	87.23	"	P_3
3350.98	29833.5	1	3351.03	"	P_4
48.84	852.6	2	48.86	"	P_4
3229.26	30958.0	1	.....	.....	P_5
28.18	968.4	1	.....	.....	P_5

The isolated line λ 5165.35 is narrow and sharp; it differs, in these respects, from the lines in the series.

POTASSIUM.

Wave-length	Oscillation frequency, in vacuum	Intensity	Previous measurement	Observer	Series and number
7697	.....	Very strong	7701.92	Lehmann	P_1
7664	.....	"	7668.54	"	P_1
6939	14407	8	6938.8	K. and R.	II_3
13	462	7	11.2	"	II_3
5832.25	17141.3	6	5832.23	"	I_4
12.53	199.5	5	12.54	"	I_4
02.12	230.8	7	02.01	"	II_4
5782.74	288.1	6	5782.67	"	II_4
5359.96	18651.8	4	5359.88	"	I_5
43.38	709.6	2½	43.35	"	I_5
40.17	720.9	3	40.08	"	II_5
23.68	778.9	2	23.55	"	II_5
5112.76	19553.1	2	5112.68	"	I_6
5099.83	602.7	1	5099.64	"	II_6
97.64	611.1	1½	97.75	"	I_6
85.07	659.4	1	84.49	"	II_6
4965.61	20132.5	1	4965.5		I_7
4957	167	<1	56.8		II_7
51.46	190.1	1	52.2	"	I_7
4870	528	<1	4870.8	L. and D.	I_8
62	562	<1	63.8	"	II_8
57	583	<1	56.8	"	I_8
29	702	<1			
03	814	<1	4803.8	"	I_9
01	823	<1	.....	.....	II_9
4798	836	<1	4796.8	L. and D.	
67	972	<1			
60	21002	<1	4759.8	"	I_{10}
4642.35	21534.4	2	4642	H. and R.	
38.6	51.8	<1			
4047.39	24700.3	9	4047.36	K. and R.	P_2
44.33	719.0	10	44.29	"	P_2
3447.56	28997.8	3	3447.49	"	P_3
06.55	29006.3	4	06.49	"	P_3
Present	.....	<1	3217.76	"	P_4
3217.36	31072.7	2	00.27	"	P_4

Other lines, very feeble indeed, appear on the strong continuous spectrum in the region near 4642. The line λ4642.35 was first observed in the spectrum of the Bessemer flame; Hartley and Ramage, *Phil. Trans.*, **196**, 491, 1901.

have been sorted into the principal, and the first and second subordinate series, and marked P, I, or II, with the number of the line, according to Rydberg's formula, in the sixth column. The

wave-lengths of the lines which have been observed before are given in the fourth column.

In the column of observers, L. and D. represent Liveing and Dewar; K. and R., Kayser and Runge; L. de B., Lecoq de Boisbaudran.

Diagrams of these spectra were drawn, as described in my former paper, to scales of oscillation frequencies for abscissæ, and (1) atomic masses, (2) squares of atomic masses for ordinates. The conclusions previously deduced from the less complete data were thereby amply confirmed. There is undoubtedly a very close connection between the spectra and the atomic masses; and the lines, which connect the corresponding members of homologous doublets in diagram (2), do intersect on the line of zero atomic mass.

The two limits in each spectrum toward which the two subordinate series appeared to converge were determined by a slight modification of Rydberg's method combined with graphical methods. These were inserted in the diagrams, and curves were drawn through the points. In diagram (1) the curves were turned away from each other and the points of bisection of the lines between the limits lay on a straight line; so also did the points of bisection of the lines between the two more refrangible and corresponding doublets of the second subordinate series. In diagram (2) the curves through the limits of the series, when produced, intersected on the line of zero atomic mass. This fact indicates that the difference between the two limits of the series, while not proportional to the square of the atomic mass, is a simple function of it. Rydberg, Kayser and Runge, and Rummel[1] have each shown that the differences between the convergence points of the subordinate series are approximately proportional to the squares of the atomic masses.

A diagram of the spectra and limits of the series was also drawn for the three metals to scales of wave-lengths and atomic masses. The more refrangible limits of the subordinate series and the more refrangible members of the second series now lay

[1] *Proc. Roy. Soc. Victoria*, 9 and 10, 1897.

on straight lines; the change in wave-length was thus proportional to the atomic mass.

After a careful study of the facts and many computations, it was found possible to calculate the subordinate series with considerable accuracy by the following formulæ:

THE FIRST SUBORDINATE SERIES.

The two convergence points (n_∞) of this series are obtained as follows: $n_\infty = 22830 - 21.633W \pm \frac{A}{2}$, where W is atomic mass and A is the average difference between the doublets. The latter quantity, as determined from the lines which are best suited for accurate measurement, is for potassium 57.8, for rubidium 236.4, and for cæsium 547.6. These values, as shown above, are simple functions of the atomic mass; but the best method of expressing them is not yet clear. This formula gives the following values for n_∞ belonging to the doublets of the first subordinate series: potassium, 21953.9 and 22011.7; rubidium, 20861.8 and 21098.2; and cæsium 19677.2 and 20224.8.

When these values are substituted for n_∞ in Rydberg's formula

$$n = n_\infty - \frac{N_0}{(m+\mu)^2};$$

in which $n = 10^8 \lambda^{-1}$, $N_0 = 109675$, $m = 3 \cdot 4 \cdot 5 \ldots$ and when we also substitute for μ (assuming it to have a constant value for the series) the value

$$\mu = 0.7869 - 1466 W^2 \times 10^{-8},$$

we obtain the following results:

POTASSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	.....	14214.6	.....	3	.....	14272.4	.....
4	17141.3	17122.4	−18.9	4	17199.5	17180.2	−19.3
5	18651.8	18653.3	+ 1.5	5	18709.6	18711.1	+ 1.5
6	19553.1	19557.1	+ 4.0	6	19611.1	19614.9	+ 3.8
7	20132.5	20134.7	+ 2.2	7	20190.1	20192.5	+ 2.4
8	20528	20526.1	− 1.9	8	20583	20583.9	+ 0.9
9	20814	20803.6	−10.4	9	.....	.....	.....
10	21002	21007.4	− 5.4	10	.....	.....	.....

RUBIDIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	.....	12763.2	....	3	.....	12999.6	....
4	15870.5	15854.4	−16.1	4	16106.8	16090.8	−16.0
5	17463.2	17462.3	− 0.9	5	17699.6	17698.7	− 0.9
6	18403.9	18404.0	+ 0.1	6	18640.3	18640.4	+ 0.1
7	19004.0	19002.3	− 1.7	7	19240.7	19238.7	− 2.0
8	19407.2	19406.1	− 1.1	8	19642.5	19642.5	0
9	19693.6	19691.3	− 2.3	9	19926	19927.7	+ 1.7
10	19902	19900.3	− 1.7	..	.....	.....	....
11	20062	20057.9	− 4.1	..	.....	.....	....

CÆSIUM.

m	Oscillation Frequencies		Differences	m	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	10852.1	10865.7	+13.6	3	11404.1	11413.3	+9.2
4	14335	14327.9	− 7.1	4	14873	14875.5	+2.5
5	16089.7	16088.2	− 1.5	5	16632.4	16635.8	+3.4
6	17102.7	17103.6	+ 0.9	6	17649.7	17651.2	+1.5
7	17739.6	17741.9	+ 2.3	7	18289.2	18289.5	+0.3
8	18166.2	18169.2	+ 3.0	8	18717.0	18716.8	−0.2
9	18463.8	18469.1	+ 5.3	9	19016.8	19016.7	−0.1
10	18682	18687.7	+ 5.7	10	19228	19235.3	+7.3
11	18848	18851.9	+ 3.9	11	19396	19399.5	+3.5

THE SECOND SUBORDINATE SERIES.

In this series

$$n_\infty = 22850 - 21.812\,W \pm \frac{B}{2},$$

where B is, for potassium, 57.8; for rubidium, 238.0; and for cæsium, 553.6; and

$$\mu = 0.7990 + 7984\,W^2 \times 10^{-9}.$$

It will be observed that the doublets in the second subordinate series are more widely separated than those in the first series. It would appear also that the two series do not converge toward the same limit; the difference between the limits, however, diminishes in the different metals as the atomic mass increases. This is true on the supposition that μ is constant, and

not variable, as in the formula given for the principal series. Kayser and Runge hold the view that there are different limits for the two series, while both Rydberg and Rummel favor the view that the limits are the same.

The observed and calculated oscillation frequencies are as follows:

POTASSIUM.

m	Oscillation Frequencies		Differences	*m*	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	14407	14417.4	+ 10.4	3	14462	14475.2	+ 13.2
4	17230.8	17229.9	− 0.9	4	17288.1	17287.7	− 0.4
5	18720.9	18720.3	− 0.6	5	18778.9	18778.1	− 0.8
6	19602.7	19603.9	+ 1.2	6	19659.4	19661.7	+ 2.3
7	• 20167	20170.5	+ 3.5	.	.....	.....	.....
8	20562	20555.3	− 6.7	.	.....	.....	.....
9	20823	20828.6	+ 5.6	.	.....	.....	.....

RUBIDIUM.

m	Oscillation Frequencies		Differences	*m*	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	13498	13496.7	− 1.3	3	13738	13734.7	− 3.3
4	16228.9	16219.6	− 9.3	4	16466.8	16457.6	− 9.2
5	17680.9	17671.4	− 9.5	5	17918.1	17909.4	− 8.7
6	18543.0	18535.8	− 7.2	6	18781.5	18773.8	− 7.7
7	19098	19091.8	− 6.2	.	.....	.....	.....
8	19480	19470.3	− 9.7	.	.....	.....	.....

CÆSIUM.

m	Oscillation Frequencies		Differences	*m*	Oscillation Frequencies		Differences
	Observed	Calculated			Observed	Calculated	
3	.....	12609.3	.....	3	.....	13162.9	.....
4	15171	15180.2	+ 9.2	4	15733	15733.8	+ 0.8
5	16566.7	16566.1	− 0.6	5	17120.2	17119.7	− 0.5
6	17397.2	17397.3	+ 0.1	6	17951.6	17950.9	− 0.7
7	17933.9	17934.7	+ 0.8	7	18487.3	18488.3	+ 1.0

The convergence points of the series as deduced in different ways are given in the following table:

Element		From above Formulæ			By calculation from observed lines, p. .	From formula for principal series [1]	Numbers calculated by Rydberg
		First Series	Second Series	Mean of two series			
					Mean		
Potassium	(1)	21953.9	21968.0	21960.95	21960	21969.4	21955.46
"	(2)	22011.7	22025.8	22018.75	22018	22024.3	22013.31
Rubidium	(1)	20861.8	20868.3	20865.65	20865	20868.6	20869.15
"	(2)	21098.2	21106.3	21102.25	22101	21112.3	21098.83
Cæsium	(1)	19677.2	19674.2	19675.7	19672	19686.7	
"	(2)	20224.8	20228.0	20226.4	20226	20234.2	

The numbers in the sixth column were obtained by the law, discovered by Rydberg and independently by Schuster, which connects the principal and subordinate series: the convergence points of the subordinate series are given by the differences between the convergence points and first lines (for which $m = 1$) of the principal series. One set of the numbers was obtained from the expression

$$\frac{N_0}{(1 + 1.19126 + 0.00103W)^2},$$

and the other set from the expression

$$\frac{N_0}{(1 + 1.19126 + 0.00103W + 182W^2 \times 10^{-8})^2}.$$

The figures in this column agree best with those of the second subordinate series in the third column; and it will be remarked, as confirming the closer connection between the principal and second subordinate series, that the results calculated for the latter series of rubidium differ by about nine units, whereas those given by the formula for the principal series differ by about 27.5 units from the observed numbers. The connection between the first subordinate series and the atomic mass is apparently simpler than between the other two series and the atomic mass.

The numbers in the last column were taken from Rydberg's [2]

[1] Author, *loc. cit.* [2] *Paris Congress Reports*, 2, 212, 1900.

paper. He calculated them by means of an empirical formula from the observed lines.

All the strong lines and nearly all the weak lines which have been observed in the flame and arc spectra of these elements, are included in the three harmonic series of lines. The empirical formulæ given show that the differences in the corresponding series depend wholly on the atomic masses of the three elements.

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Minor Contributions and Notes.

A DETERMINATION OF THE WAVE-LENGTHS OF THE BRIGHTER NEBULAR LINES.[1]

In the course of a recent investigation of the spectrum of *Nova Persei* (*Bulletin* No. 8) there was developed the necessity for a more accurate knowledge of the wave-lengths of the brighter nebular lines than then existed. The wave-lengths of the two chief nebular lines had been determined with great accuracy by Dr. Keeler in his classical work on the motions in the line of sight of the brighter gaseous nebulæ,[2] but the uncertainties in the positions of the remaining lines were of the order of a tenth-meter or more, according to the brightness of the line and its position in the spectrum. With a view of obtaining a more accurate knowledge of these wave-lengths, the spectra of a number of the brighter nebulæ have been photographed, and the results are given in the following pages.

Table I contains a list of the negatives secured, and some of the details of the exposures. The first column contains the plate number; the second, the name of the nebula whose spectrum was photographed; the third, the Pacific Standard Time of the middle of the exposure, and the fourth, the length of exposure. The letter in the fifth column refers to the spectrograph employed, as described below. The sixth column specifies the region covered by the photograph, and the seventh records the elements used for comparison spectra.

These spectra were obtained with spectrographs attached to the thirty-six-inch telescope. The small photographic correcting lens was used in photographing Nos. 2204, 10, 13, 34, 36, 96, and 97. Except in the cases of very small nebulæ, the twelve-inch telescope is optically more efficient for such work, but the superior appointments of the larger instrument more than compensate for its disadvantages.

[1] *Lick Observatory, University of California, Bulletin* No. 19.

The results given in this paper were presented before the December meeting of the Astronomical and Astrophysical Society of America.

[2] *Publications of the Lick Observatory*, III.

TABLE I.

Plate No.	Object	Date	Exp. time	Inst	Region of spectrum	Comparison
2202F	*G. C.* 4390	1901 July 22 10^h	3^h 00^m	*b*	λ 4000—λ 6000	*He*
04F	*G. C.* 4390	24 10	3 10	*b*	3700— 5700	*Fe, H, He*
10C	*N. G. C.* 7027	Aug. 2 13	7 10	*b*	3700— 5700	*Fe, H, He*
13D	*G. C.* 4390	5 11	3 37	*b*	3700— 5700	*Fe, H, He*
22D	*N. G. C.* 7027	9 10	3 59	*b*	4000— 6000	*He*
34C	Ring Nebula in *Lyra*	17 12	3 40	*b*	3700— 5700	*Fe, He*
36C	*G. C.* 4964	18 13	7 20	*b*	3700— 5700	*Fe, He, H*
68E	*N. G. C.* 7027	Sept. 15 8	2 04	*c*	4700— 5100	*He, H*
96B	*G. C.* 4964	Nov. 4 11	3 20	*b*	3700— 5700	*He, H*
97E	*Orion* Nebula	4 15	4 00	*b*	3700— 5700	*He, H*
2313B	*Orion* Nebula	Dec. 8 12	5 02	*a*	4200— 4500	*Fe, H*
14A	*Orion* Nebula	12 12	4 30	*b*	3700— 5700	*Fe, He, H*
15E	*Orion* Nebula	16 12	4 40	*c*	4700— 5100	*Fe, He, H*
16A	*Orion* Nebula	17 12	4 30	*c*	4700— 5100	*Fe, Pb, He, H*
2317A	*Orion* Nebula	18 12	4 20	*a*	4200— 4500	*Fe, H*

The spectrographs used were as follows:

a. The Mills spectrograph.

Focal length of collimator lens, 722.4 mm; three prisms; focal length of camera lens, 405.5 mm. Collimator and camera corrected for $H\gamma$.

b. A single light flint prism instrument into which the Mills spectrograph is convertible, and which has been described in *Bulletin* No. 8 as "Spectrograph No. I." The lenses are the same as those mentioned above.

c. A spectrograph containing three dense flint prisms, similar to spectrographs III and IV described in *Bulletin* No. 8, except that the prisms were set at minimum deviation for λ 4950, which ray was brought to the center of the field. Plate No. 2268 was secured with the regular camera lens referred to above, and Nos. 2315E and 2316A with a lens of the same focal length corrected for λ 5900. In using the regular camera lens the plate-holder had to be tilted, but the combination of photographically corrected collimator and visually corrected camera gives in this region a field at right angles to the line of collimation.

The range of spectrum for the one prism instrument is normally from above λ 3700 to λ 5000. By slightly tilting the plate-holder equally good definition is afforded between λ 4000 and λ 6000. Two

exposures were made with the latter range for the purpose of photographing the nebular lines below λ 5007. They were unsuccessful, as no trace of radiation below the chief nebular line was secured. On account of the unfavorable positions of the comparison lines recorded upon them, neither of these plates was used to determine the wave-lengths of the lines actually photographed. The one spectrum of the Ring nebula secured (No. 2234C) was very weak, only the two nebular lines at λ 4959 and λ 5007 being recorded. It was accordingly not used. One of the plates of *G. C.* 4964 was also underexposed and rather poor. Values of wave-lengths determined from this nebula were therefore given weight one in making up the final list. In Table II will be found the results of measurements of the other plates secured with this instrument.

TABLE II.

Orion nebula 2 plates	*i*	*G. C.* 4390 2 plates	*i*	*G. C.* 4964 2 plates	*i*	*N. G. C.* 7027 1 plate	*i*
3726.35[1]	4						
3728.98[1]	3						
3835.8	1—						
3868.87	4	3868.94	5	3868.88	4+	3868.97	7
3889.13	4	3889.16	2				
3965.1	1—						
3967.69	3	3967.64	5	3967.57	4	3967.61	7
3970.24	5	3970.25	4	3970.08	2	3970.33	3
4026.7	1	4026.8	1—				
4068.7	1					4068.81	2
4101.95	6	4101.90	6	4101.88	4	4101.88	7
4340.60	10	4340.64	7	4340.67	6	4340.60	9
		4363.30	2	4363.5	2	4363.37	8
4471.77	2	4471.66	2			4471.7	1—
				4685.72	6	4685.75	9
						4740.0	1
4861.57	7	4861.47	6	4861.7	4	4861.44	8
4959.01	5	4959.13	8	4959.16	8	4958.97	20
5006.82	7	5006.94	25	5007.06	20	5006.83	80

The results given above are corrected for velocity in the line of sight. In making this correction the velocity for which the wave-lengths of the lines on any plate were corrected was determined from the hydrogen lines of that plate. This method of procedure was

[1] These wave-lengths depend upon the measurements of only one plate. In the first plate secured there were no comparison lines sufficiently close to them to give accurate determinations.

adopted for the purpose of eliminating any *error of the plate*, or relative shift of nebular and comparison lines which might result from a number of causes, such, for instance, as temperature changes and differential flexure during the exposures, which were necessarily long. In order to reduce any such effects to a minimum the comparison spectra were introduced at frequent intervals during the exposure. These precautions were taken even though with the longest exposures the definition was practically perfect. The correction for velocity was also applied to the wave-lengths of the hydrogen lines themselves. Had the negatives been somewhat stronger, velocity determinations could also have been made from a number of the helium lines.

The relative brightnesses of the lines as photographed are indicated in a way by the numbers opposite their wave-lengths. About all that can be said of the system is that for any one object, the larger the number, the brighter the line. Such estimates must of course be used with caution, as the apparent brightness of a line is greatly influenced by photographic and instrumental conditions.

Special interest attaches to the doublet at λ 3969. This has heretofore been observed as a single line and identified as $H\epsilon$, which is in fact its least refrangible component. The two components vary much in relative intensity in different nebulæ. In *N. G. C.* 7027, λ 3967.65 is much the stronger of the two, while in the case of the *Orion* nebula, in which the hydrogen series is relatively bright as compared with the chief nebular lines, it is much the fainter. In the *Orion* nebula there is an additional line at λ 3965 (helium).

The doublet at λ 3727 has heretofore always been observed as single.

The velocities in the line of sight determined from the low dispersion spectrograms are liable to an error of perhaps five kilometers a second. They are as follows:

Object	Velocity in line of sight
G. C. 4390	− 11 km per sec.
G. C. 4964	− 7
N. G. C. 7027	+ 6
Orion nebula	+ 17

The substances whose spectra were used for comparison purposes in these low dispersion spectrograms are iron (spark), helium, and hydrogen. The iron wave-lengths have been taken from Kayser's

paper[1] whenever possible, and otherwise from Rowland's solar spectrum wave-lengths. In the case of the iron line near λ 4958 a special investigation was necessitated by the fact that the line is double and too close to be resolved by the instruments used. The position of the line was measured on two plates secured with instrument *c* as used during the observations of nebulæ, employing as reference lines those at λλ 4941.77, 4948.40, 4964.90, 4968.75, and 4973.25 in the titanium spectrum. The values determined from the plate are:

λ 4957.77
4957.80

Mean 4957.78

This result confirms that of Dr. Hartmann,[2] who found the doublet to consist (in the spark spectrum) almost entirely of the least refrangible component, the wave-length of which is λ 4957.78. In this connection it may be stated that the coil used for our comparison spectra gives a spark of about an inch without a condenser in the secondary circuit. When a metallic spark is employed, a condenser of about 0.003 microfarads is placed in the secondary circuit. The wave-lengths of the helium lines are those determined by Runge and Paschen.[3] The hydrogen wave-lengths (see Table III) are, with two exceptions, taken from Ames' paper.[4] In one of the high dispersion observations to be described later, the lead line at λ 5005.63 was used.

The positions given in Table II for the two lines below *Hβ* were not regarded as very trustworthy. Among the difficulties encountered in this region were the relatively low dispersion and the fact that the distance from the ray of minimum deviation was such as to make the determination of wave-lengths somewhat uncertain. It was for the purpose of securing more accurate determinations of the wave-lengths of the lines in question that the high dispersion spectrograms were made. The radial velocities determined from them are as follows:

[1] *Annalen der Physik* (4) **3**, 1900; also ASTROPHYSICAL JOURNAL, **13**, 329, 1901.

[2] *Sitz. der K. Akad.*, Berlin, **12**, 237, 1902; ASTROPHYSICAL JOURNAL, **15**, 292, 1902.

[3] *Sitz. der K. Akad.*, Berlin, July, 1895, 639 and 759.

[4] *Phil. Mag.*, July, 1890.

ORION NEBULA

Date		Line	Velocity[1]	Wt
1901 Dec.	8	$H\gamma$	+ 17.1 km	3
	16	$H\beta$	+ 16.1	2
	17	$H\beta$	+ 17.0	2
	18	$H\gamma$	+ 14.8	3

Mean + 16.2 km ± 0.4 km per sec.

N. G. C. 7027.

Date		Line	Velocity
1901	Sept. 15	$H\beta$	+ 12.3 km per sec.

In the case of the *Orion* nebula the wave-lengths for the two chief nebular lines were corrected for the mean velocity given above. The resulting wave-lengths are:

ORION NEBULA		*N. G. C.* 7027	
λ	weight	λ	weight
4959.07 ± 0.05	2	4958.95	½
5006.91 ± 0.05		5006.80	

In determining these wave-lengths an interpolation curve was passed through λλ 4861.50, 4922.10, and 5015.73. The iron line λ 4957.78 is recorded on both plates of the *Orion* nebula. In both cases the computed value of the wave-lengths is λ 4957.73. A compromise correction of + 0.03 was therefore added to the value of the second line as determined from the interpolation curve. In the case of the nebula *N. G. C.* 7027 the iron spectrum was not used, and the value given is that determined by interpolation.

Column one of Table III gives the finally adopted values of the wave-lengths. The values of the last two depend only upon the high dispersion observations. The values determined with instrument *b* are λ 4959.07 and λ 5006.90.

TABLE III.

Nebulæ	Vacuum tubes	Nebulæ	Vacuum tubes
3726.4 ±0.2		4101.91	4101.89 $H\delta$
3729.0 ± .2		4340.62	4340.63 $H\gamma$
3835.8 ± .5	3835.6 $H\eta$	4363.37±0.10	
3868.88± .05		4471.71±0.10	4471.65 helium
3889.14	3889.15 $H\zeta$	4685.73±0.10	
3965.1 ± .4	3964.9 helium	4740.0 ±1.0	
3967.65± .05		4861.54	4861.50 $H\beta$
3970.23	3970.25 He	4959.05±0.05	
4026.7 ± .5	4026.3 helium	5006.89±0.05	
4068.8 ± .5			

[1] These velocities refer to the region immediately preceding the Trapezium.

The probable errors attached to the wave-lengths have been assigned in an arbitrary manner, but in accordance with the judgment of the observer.

For purposes of comparison the results of Keeler[1] and Hartmann[2] for the wave-lengths of the two chief nebular lines are appended. Keeler's value for the second line has been increased by 0.15 t. m. to allow for the difference in the positions of the iron doublet assumed by himself and the writer, respectively.

Keeler	Hartmann	Wright
4959.17	4959.17	4959.05
5007.05	5007.04	5006.89

The writer wishes to express his obligation to Dr. Campbell and Dr. Reese; the former for placing at his disposal the requisite instrumental equipment for the foregoing work and for valuable assistance, and the latter for great assistance rendered in the observations.

W. H. WRIGHT.

May 26, 1902.

EFFECT OF SELF-INDUCTION UPON SPARK-SPECTRA OF METALLOIDS.

THE well-known researches of Schuster and Hemsalech (reviewed in this JOURNAL, **14**, 370–371, 1901) have been recently extended by M. A. de Gramont (*Comptes Rendus*, **134**, 1048–1050, 1205–1207), who has studied the variation of intensities in spectral lines with change in amount of self-induction. The substances examined include a large number of metalloids, especially in combinations occurring in minerals.

Among the results obtained with the condensed spark we note here only three, namely:

1. In general, the lines due to metalloids disappear before the spectra of metals are sensibly affected by increasing self-induction.

2. The C line of hydrogen remains long after increasing self-induction has extinguished the air lines. The extinction of this particular line demands an inductance of at least 0.00023 henries.

3. The spectrum of carbon, even in the strongly condensed spark, is rapidly weakened by the introduction of inductance. Using 0.00002 henries all carbon lines except *Ca* (λ6579) are diminished in intensity,

[1] *Publications of the Lick Observatory*, III. [2] *Loc. cit.*

while with 0.00006 henries they disappear almost completely. The disappearance of *Ca* occurs only when the inductance reaches 0.00350 heuries.

The reader is referred to the reference above given for the effect upon the spectra of *P, Te, As, Sb, C, Si, Ge, Th,* and *Na*.

H. C.